

PHYS127AL Lecture 10

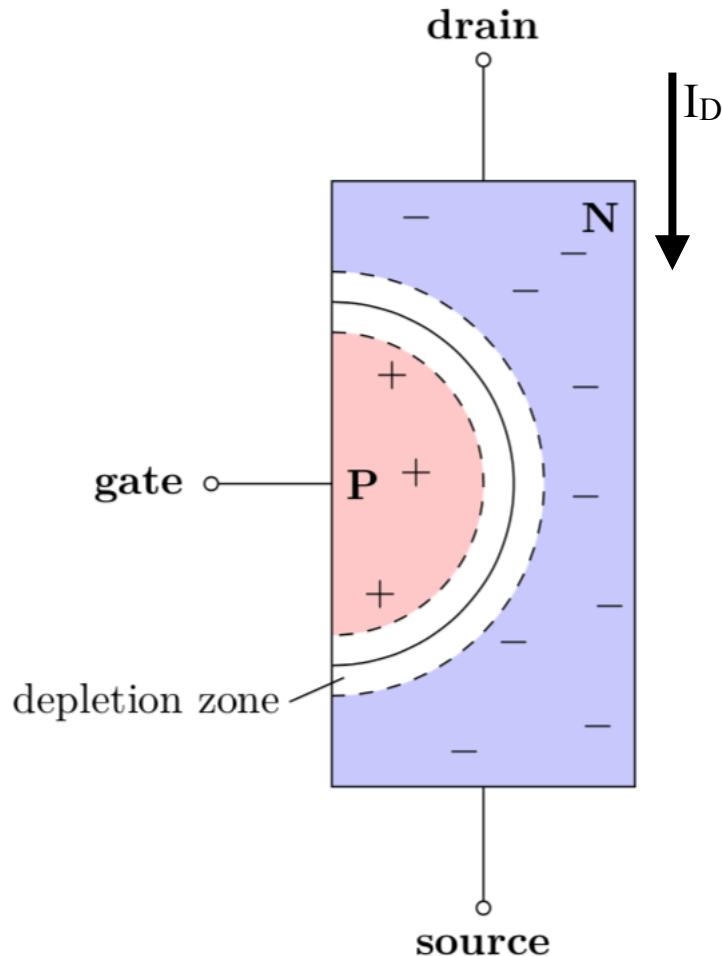
David Stuart, UC Santa Barbara

Operational Amplifiers



Review: Field effect transistors

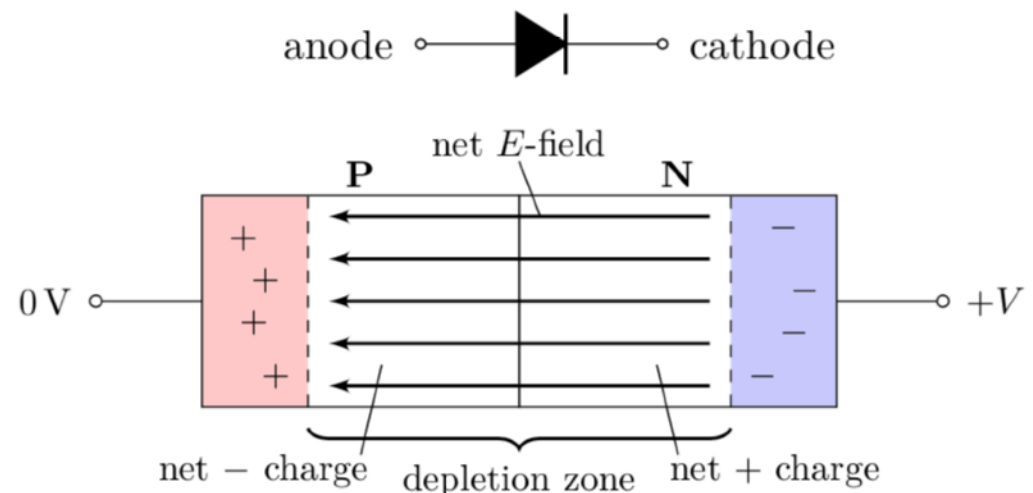
The NPN and PNP transistors we've discussed so far are called bi-polar junction transistors (BJT). FETs operate under a different mechanism.



This is a junction FET (jFET) where a p-type region is implanted within an n-type bulk. The depletion region can be controlled by the gate. Lower V_G increases the depletion.

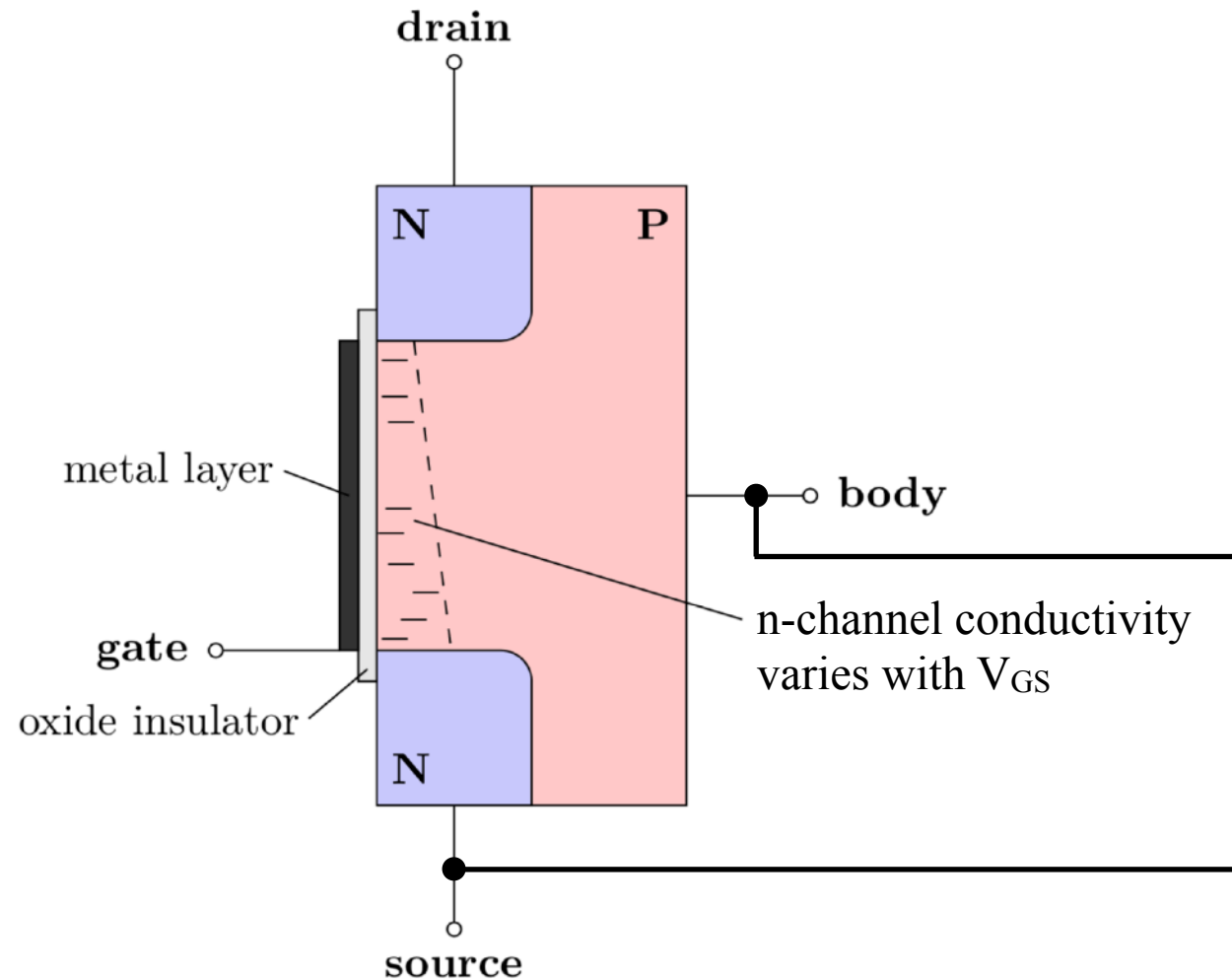
$$I = n A q v$$

So changing the gate voltage controls n and I . Like pinching off a hose. "Depletion mode"



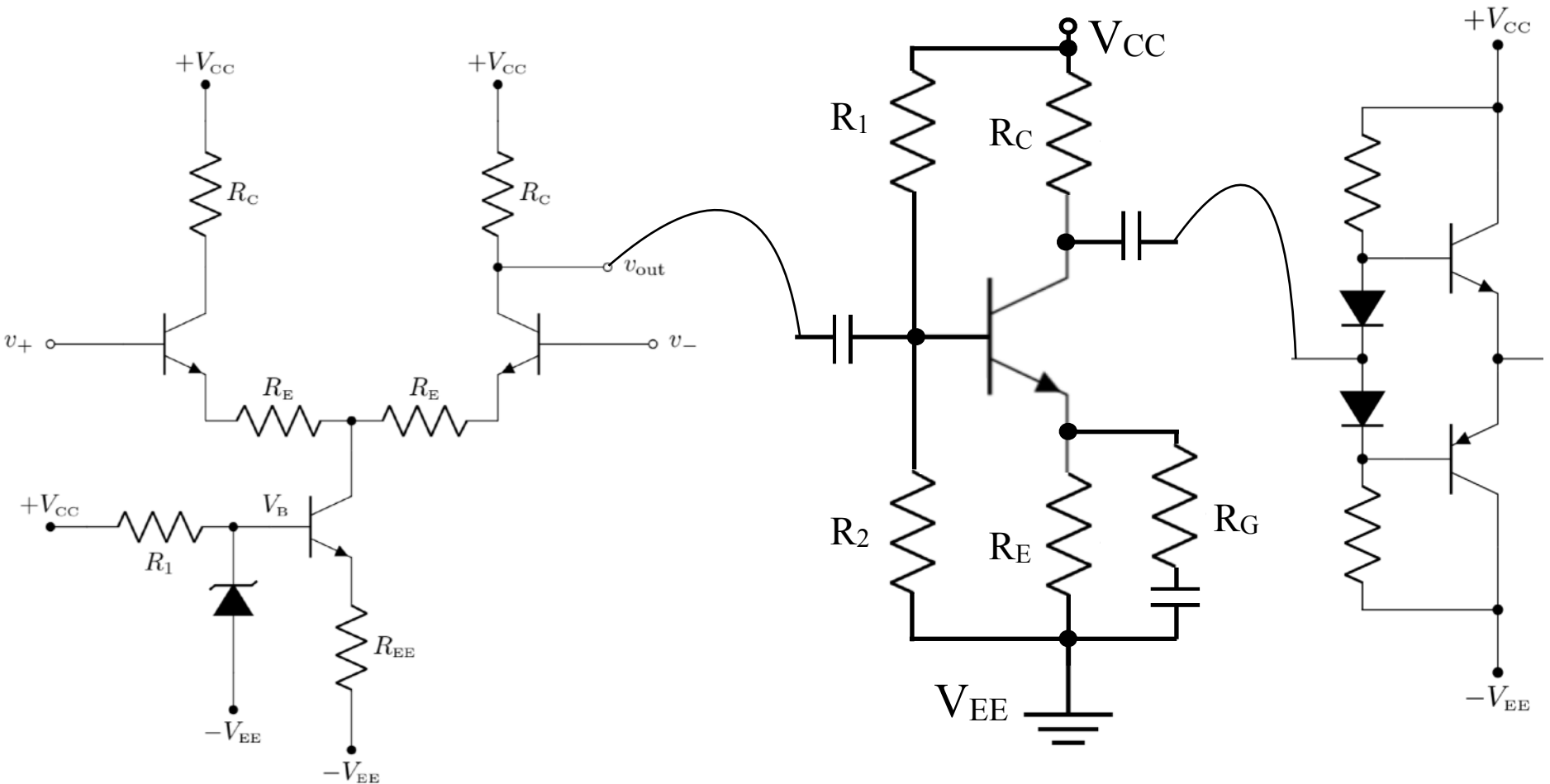
Field effect transistors (FETs)

Note that the MOSFET has large input impedance since little current flows through capacitor; just induces charge to enable I_D current flow.



Operational amplifiers

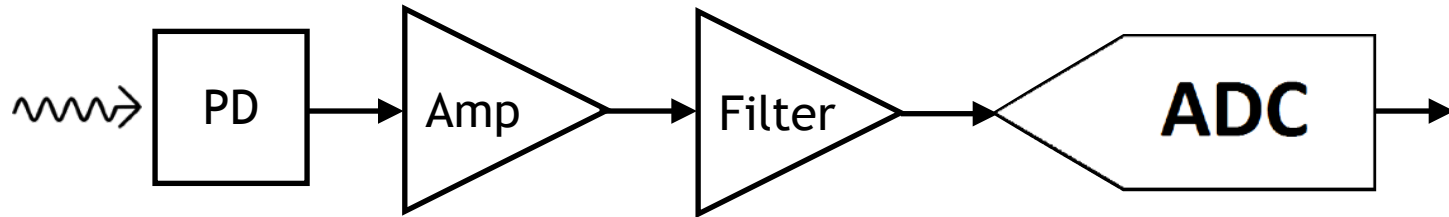
We developed a variety of transistor based circuits that can be used to amplify signals. A general purpose one would be a differential amp, where we could ground one side if needed. And, it should include all the optimization features: temperature compensation, controllable gain, high current push-pull output.



Operational amplifiers

We developed a variety of transistor based circuits that can be used to amplify signals. A general purpose one would be a differential amp, where we could ground one side if needed. And, it should include all the optimization features: temperature compensation, controllable gain, high current push-pull output.

This is the ideal tool for the amp stage in our original experiment circuit.

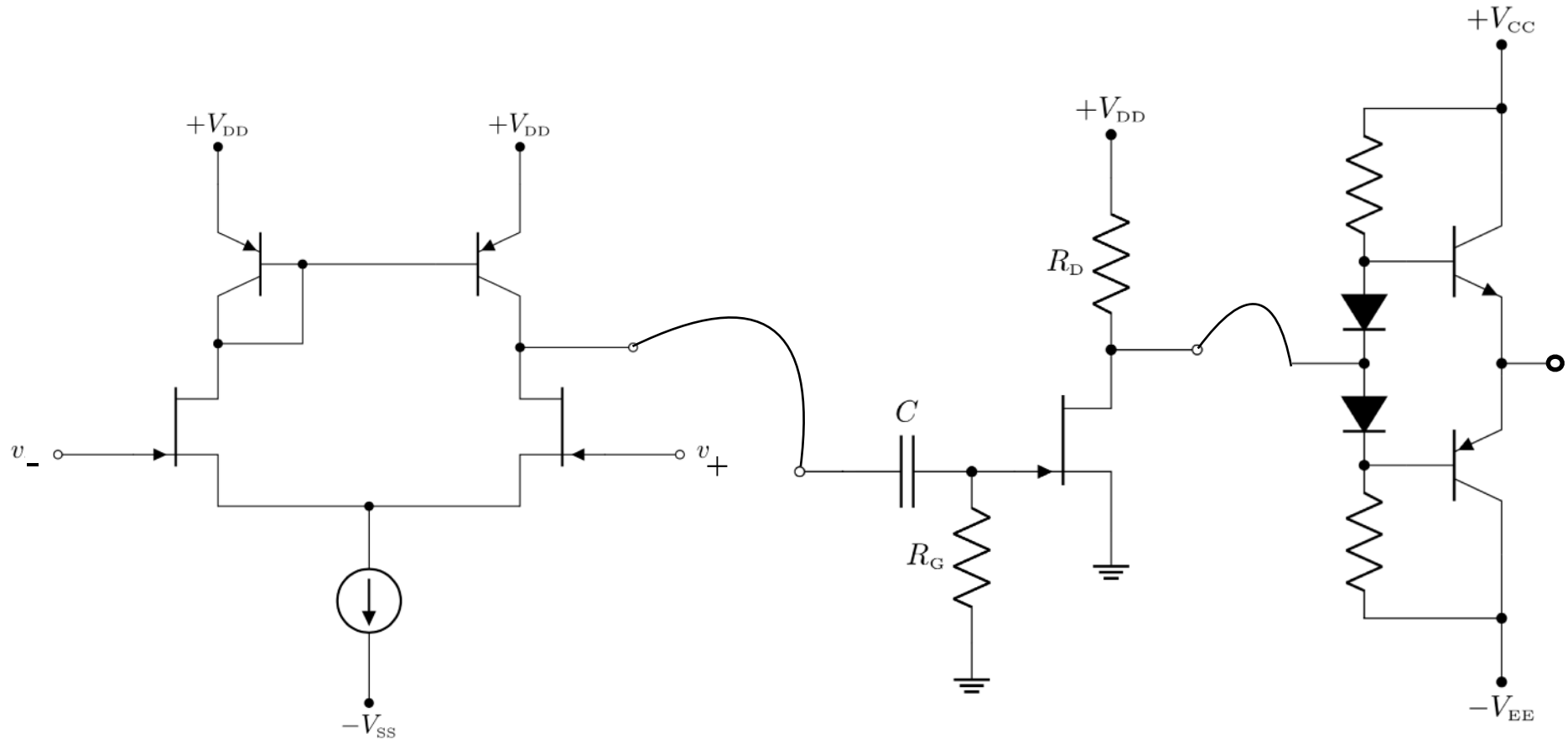


But we don't want to have to build all the details in every time. Better to have an off-the-shelf solution that incorporates all the higher order details, like temperature stability, and lets us control at a high-level. Then we can use that for all the stages with only minor configuration.

Operational amplifiers are just that — a general tool that handles the details with a simple external interface.

Operational amplifiers - internals

The (simplified) internals of an operational amplifier (op-amp) are:



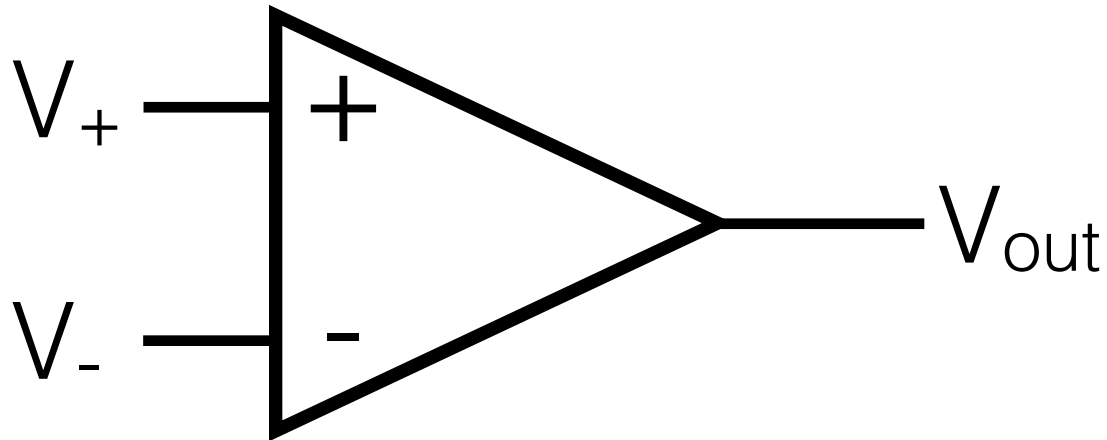
Differential amplifier with high gain and large CMMR

High gain inverting amp

High current output stage

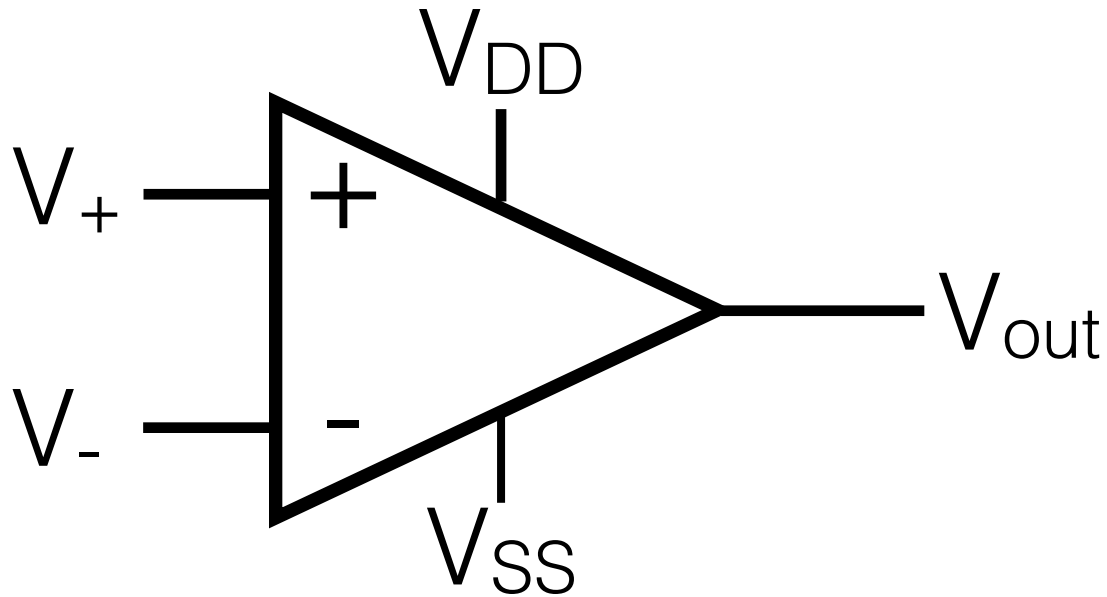
Operational amplifiers - external view

The external view of an op-amp is two inputs and one output.



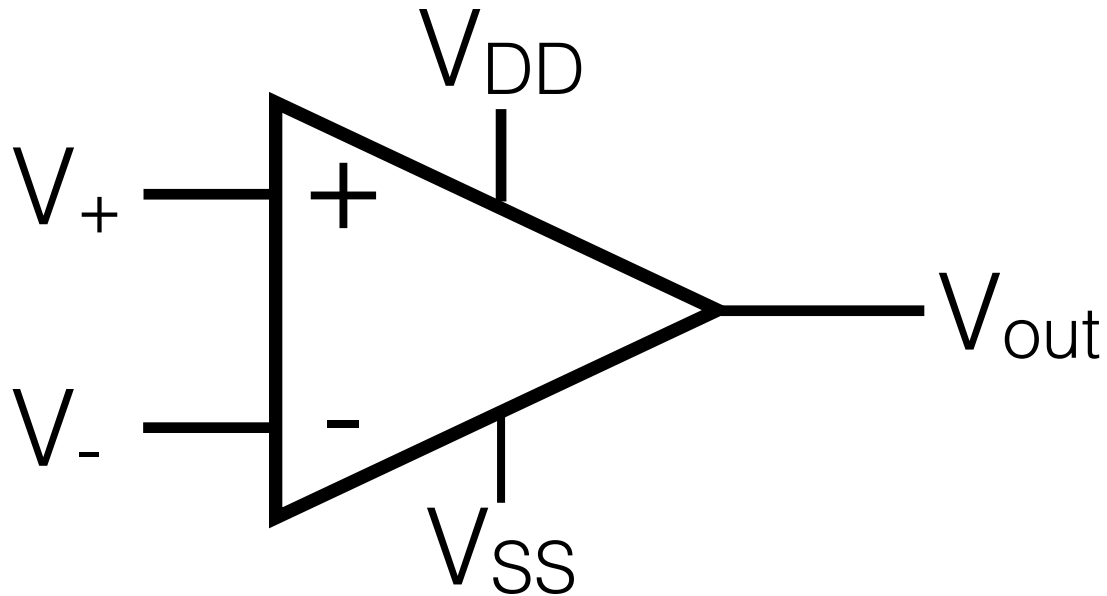
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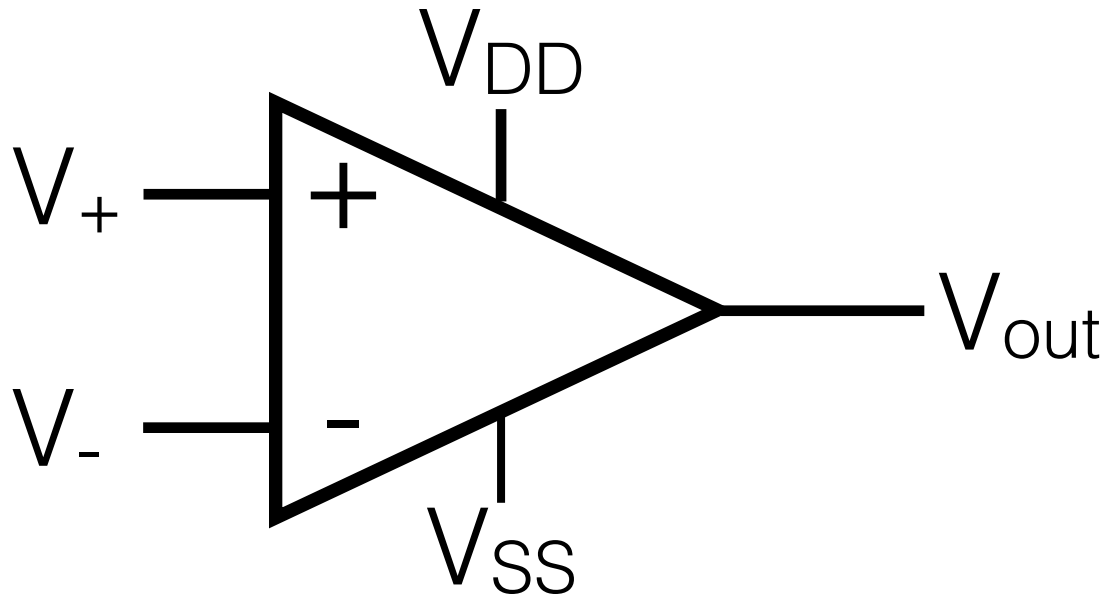
It operates as an enormous gain differential amplifier, so:

if $V_+ > V_-$ that difference is amplified to make $V_{out} = V_{DD}$.

if $V_+ < V_-$ that difference is amplified to make $V_{out} = V_{SS}$.

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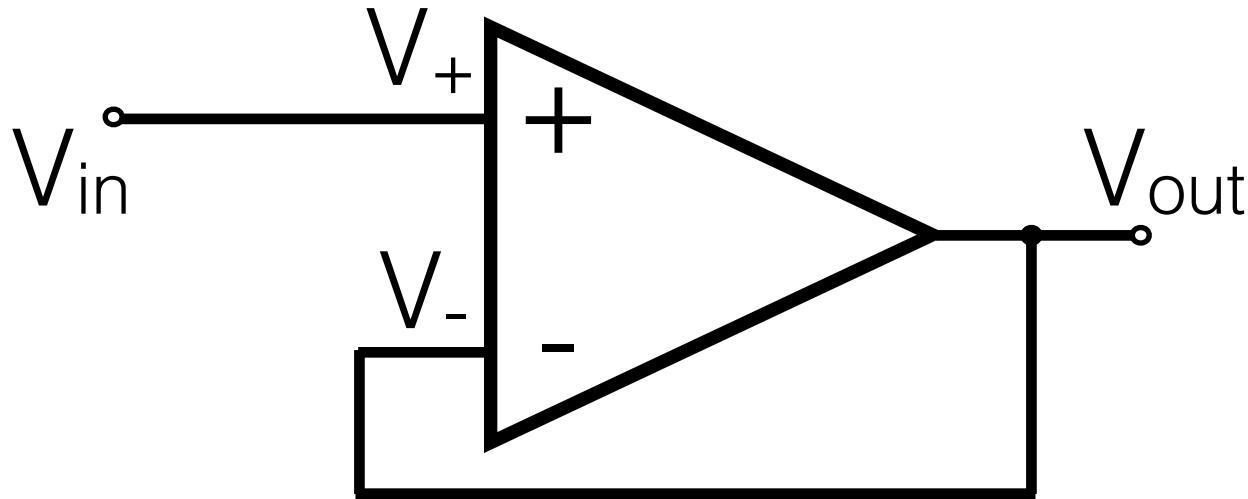
if $V_+ < V_-$ that difference is amplified to make $V_{out} = V_{SS}$.

That would be useful to compare the two inputs, but we can use it for much more using negative feedback...

Negative feedback

The versatility of op-amps comes when their enormous gain is combined with external negative feedback.

E.g., consider this circuit where we have *feedback* from the output to V_- .



If $V_{out} = V_{in}$ then $V_+ = V_-$ and there is no difference to amplify, so no change.

If $V_{in} > V_{out}$ then $V_+ > V_-$, and that positive difference is strongly amplified to rapidly increase V_{out} until it reaches V_{in} .

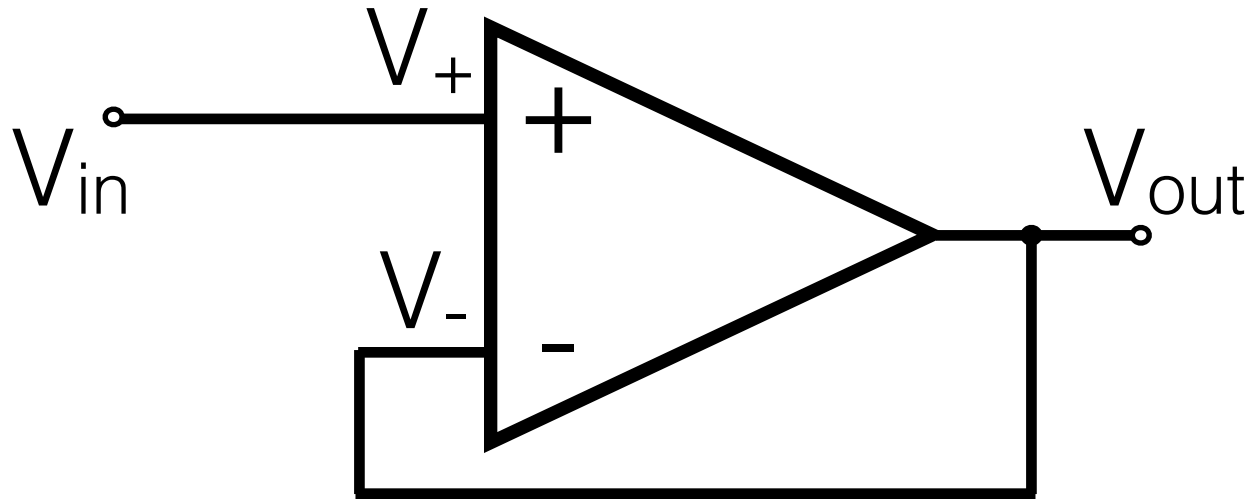
If $V_{in} < V_{out}$ then $V_+ < V_-$, and that negative difference is strongly amplified to rapidly decrease V_{out} until it reaches V_{in} .

This robustly holds V_{out} equal to V_{in} , even as V_{in} changes. \Rightarrow It's a follower.

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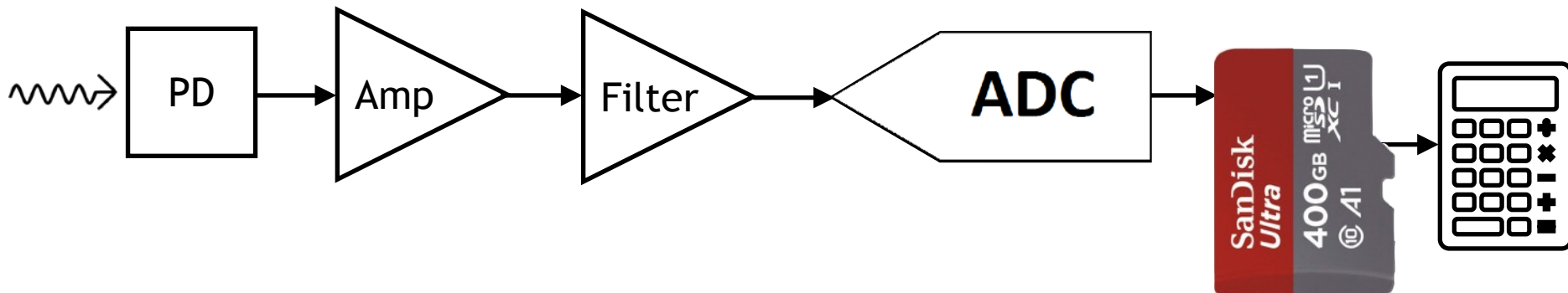
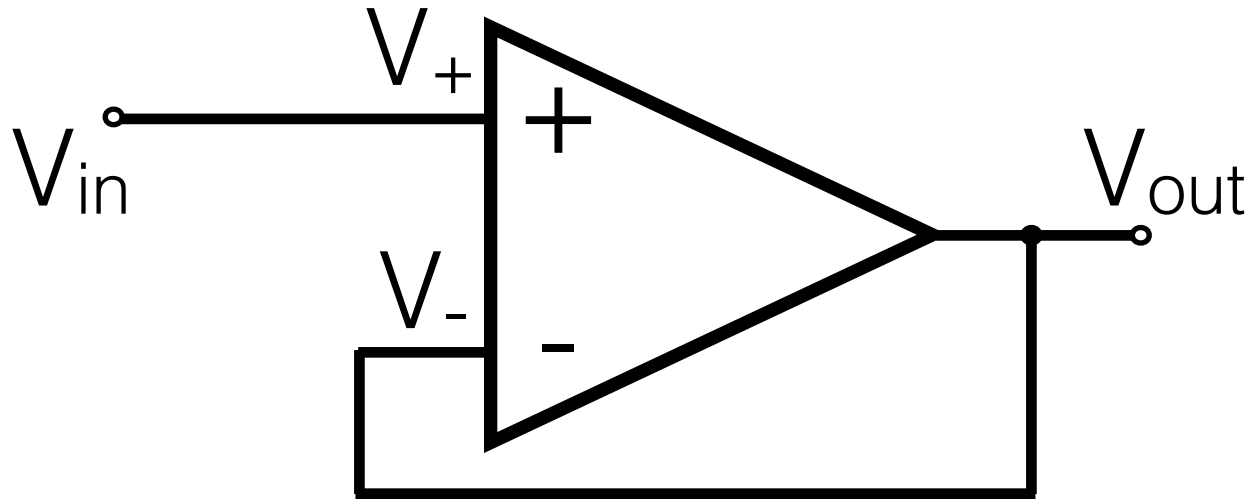
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And it has high input impedance due to FET inputs.

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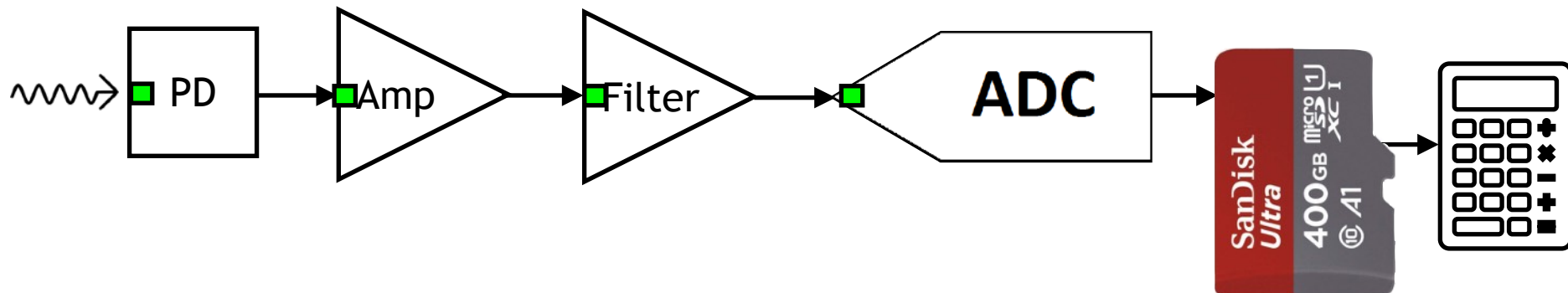
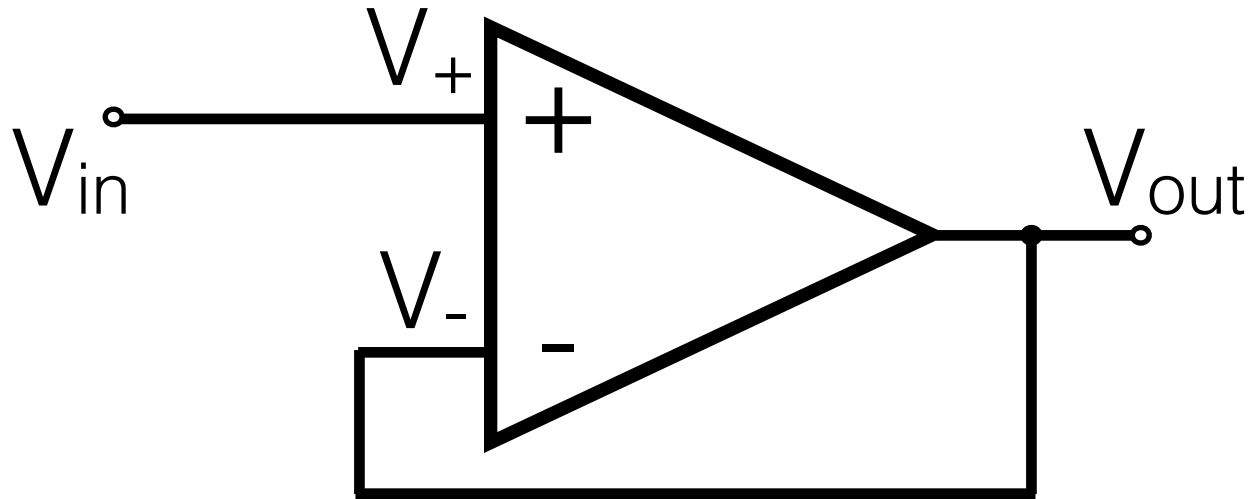


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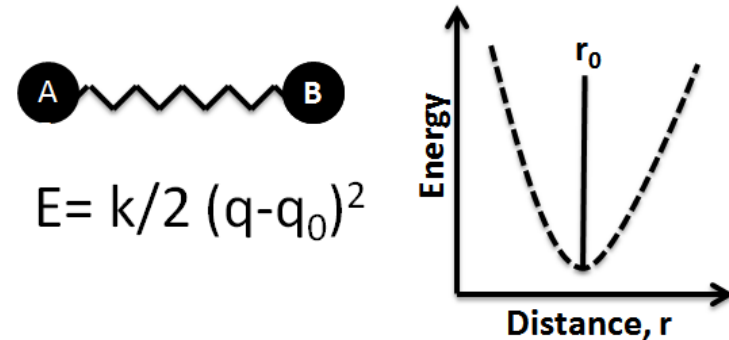
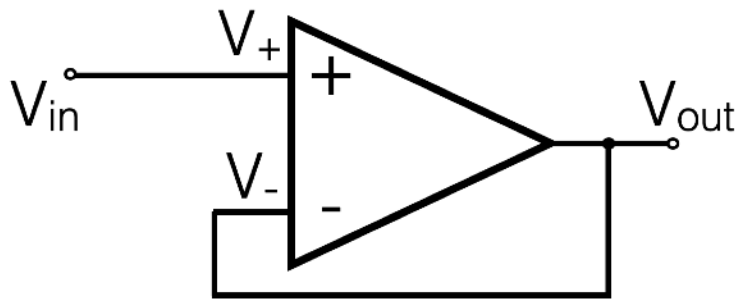
E.g., consider this circuit where we have *feedback* from the output to V_- .



And it has high input impedance due to FET inputs.

Negative feedback

This has no gain, but we traded large gain for precise control (and large X_{in}). That is the idea of negative feedback: use the large differential gain of the op-amp to allow simple external connections between the output and the inverting input (negative feedback) to define a relation between V_{out} and V_{in} .



You can think of this as defining a potential distribution; deviations from equilibrium push the system back to equilibrium.

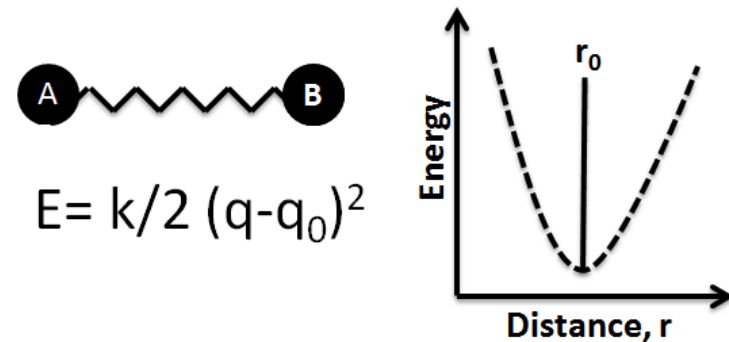
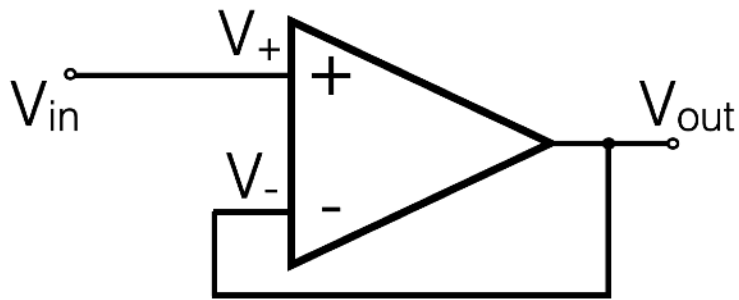
Changing V_{in} changes the equilibrium point and the op-amp readjusts.

Here the equilibrium is at $V_{out} = V_{in}$, which is $r=0$ in the spring analogy.

We'll see how to adjust the equilibrium $V_{out}(V_{in})$ relationship soon.

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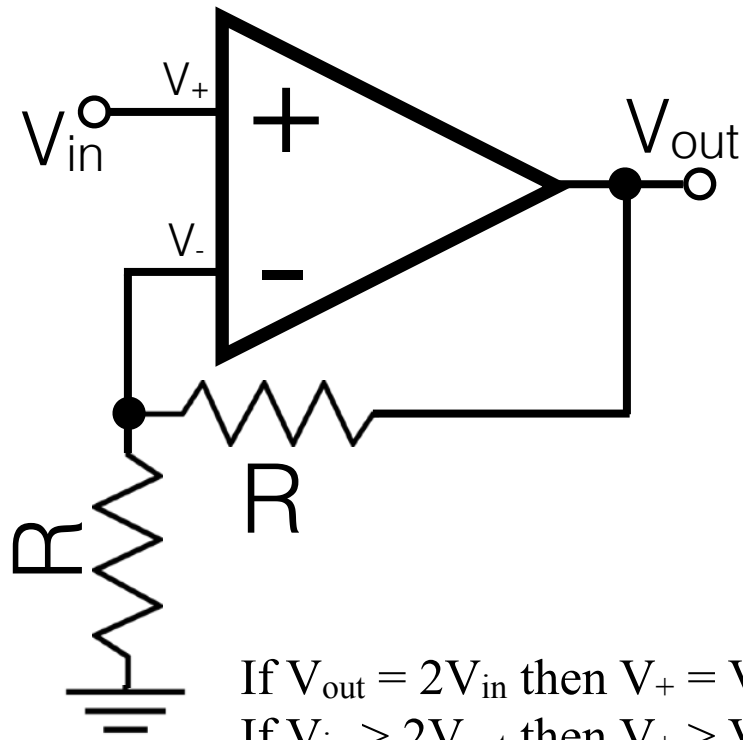
We know how it does that, by using a differential amplifier and large gain.

But, we don't need to know how; we only need to know that it does that.

Negative feedback

The versatility of op-amps comes when their enormous gain is combined with external negative feedback.

E.g., consider this circuit where we have other *feedback* from the output to V_- .



If $V_{out} = 2V_{in}$ then $V_+ = V_-$ and there is no difference to amplify, so no change.

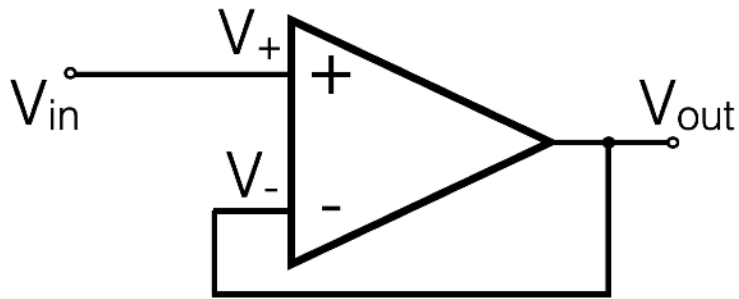
If $V_{in} > 2V_{out}$ then $V_+ > V_-$, and that positive difference is strongly amplified to rapidly increase V_{out} until it reaches $2V_{in}$.

If $V_{in} < 2V_{out}$ then $V_+ < V_-$, and that negative difference is strongly amplified to rapidly decrease V_{out} until it reaches $2V_{in}$.

This robustly holds V_{out} equal to $2V_{in}$, even as V_{in} changes. \Rightarrow It's an amplifier.

Op-amp golden rules

The “golden rules” for op-amp operation encode this idea in two simple rules that are sufficient to analyze the behavior of *most* op-amp circuits:



Golden rules:

1). No current flows into the inputs, i.e.,
 $I_+ = 0 \quad \& \quad I_- = 0$

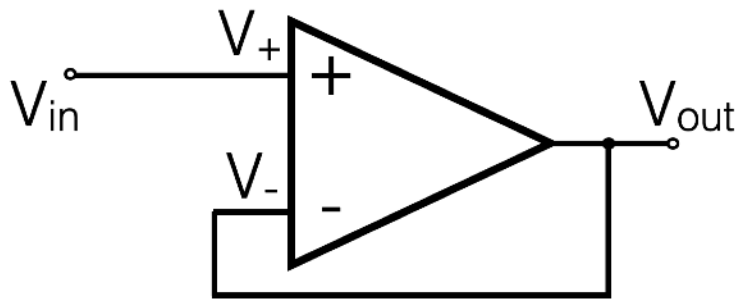
This follows from the FET inputs.

2). The op-amp output will do whatever it can to force the inputs to be equal, i.e.,
 $V_+ = V_-$

Rule 2 can only work when there is some sort of feedback from V_{out} to V_- , i.e., some negative feedback. Otherwise, V_{out} is “*at a rail*” if $V_+ \neq V_-$.

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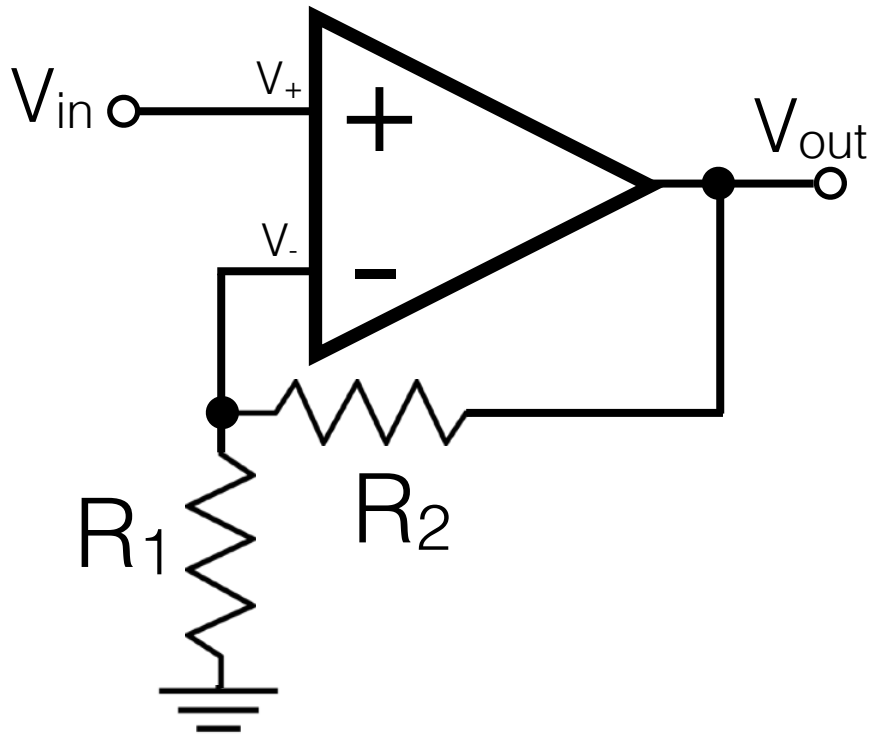
Rule 2 can only work when there is some sort of feedback from V_{out} to V_- , i.e., some negative feedback. Otherwise, V_{out} is “*at a rail*” if $V_+ \neq V_-$.

So, rule 2 immediately identifies this as a follower.

Rule 1 means it is a *high impedance* follower.

Op-amp non-inverting amplifier

Using the golden rules we can analyze the circuit for a non-inverting amplifier



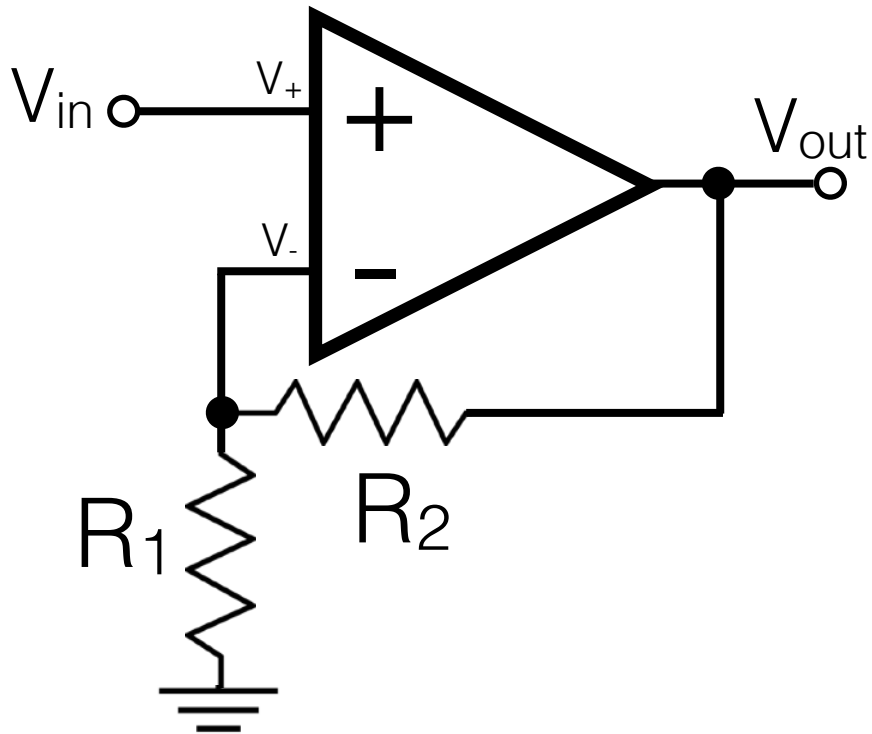
Rule 1 means high impedance.

Rule 2 means that V_{out} relates to $V_{in}=V_{+}=V_{-}$ through a simple voltage divider relationship.

$$V_{-} = V_{out} R_1 / (R_1 + R_2) = V_{in}$$

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Rule 1 means high impedance.

Rule 2 means that V_{out} relates to $V_{in}=V_+=V_-$ through a simple voltage divider relationship:

$$V_- = V_{out} \frac{R_1}{(R_1+R_2)} = V_{in}$$

Solving for V_{out} in terms of V_{in} gives

$$V_{out} = V_{in} \frac{R_1 + R_2}{R_1}$$

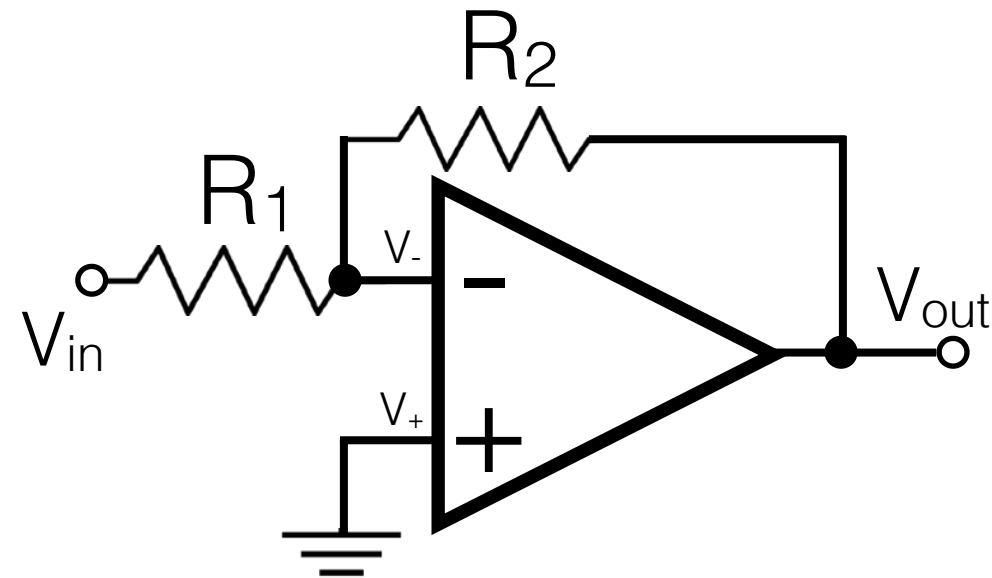
$$V_{out} = V_{in} \left(1 + \frac{R_2}{R_1} \right)$$

$$G \equiv \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}$$

Check the limits on R_1 & R_2 at 0 & ∞

Op-amp inverting amplifier

Using the golden rules we can analyze the circuit for an inverting amplifier

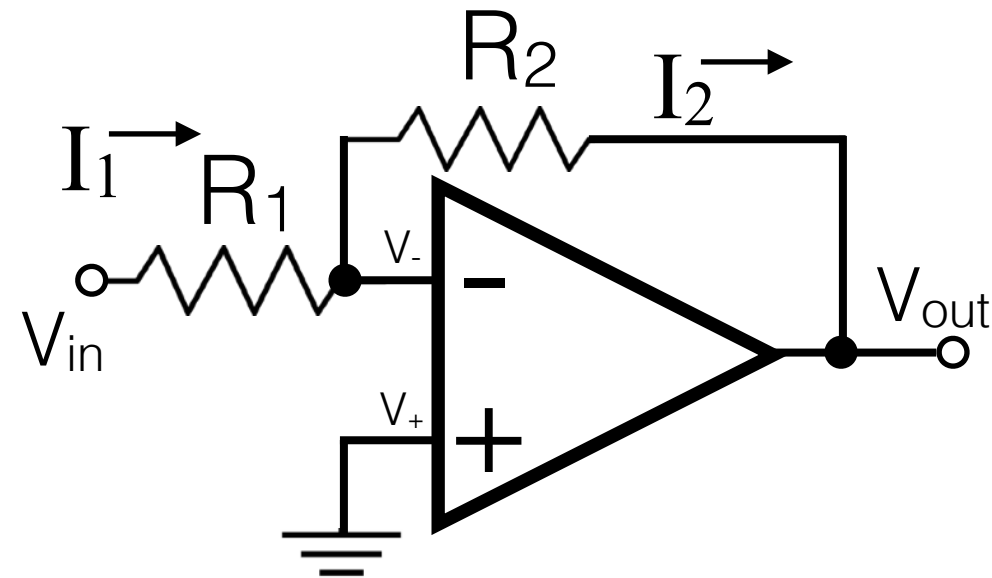


Rule 2 means $V_- = \text{ground}$.
Called a “virtual ground”.

Rule 1 means that no current flows into inverting input. So,

Op-amp inverting amplifier

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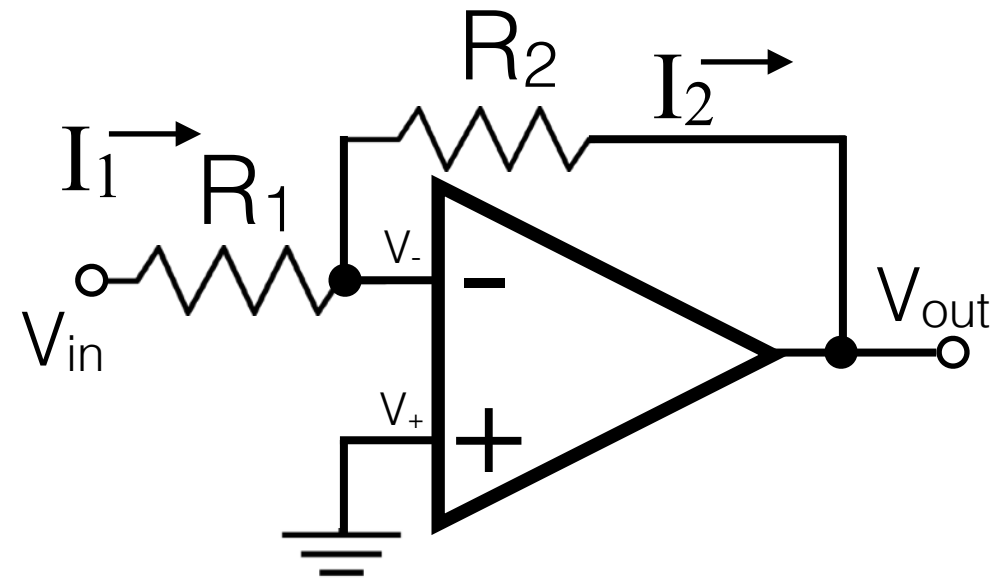


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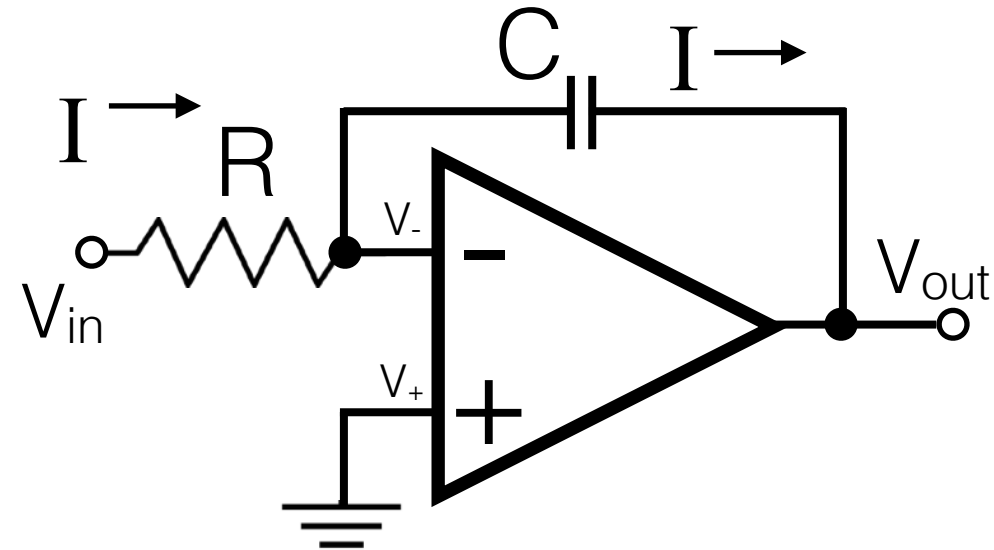
$$V_{in} = I_1 R_1$$

$$V_{out} = -I_2 R_2 = -I_1 R_2 = -V_{in} R_2 / R_1$$

$$G = -R_2 / R_1$$

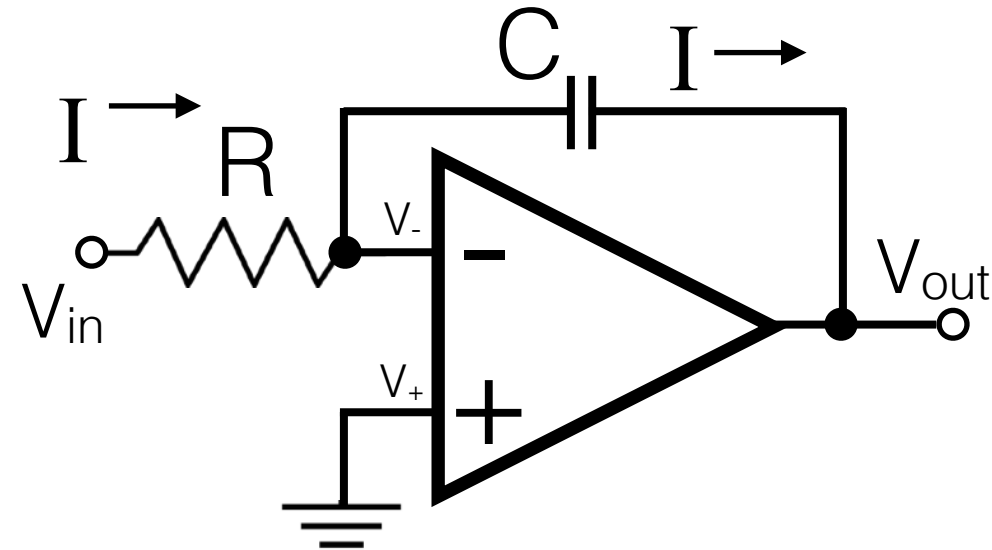
Op-amp integrator

Using the golden rules we can analyze the circuit for an integrator



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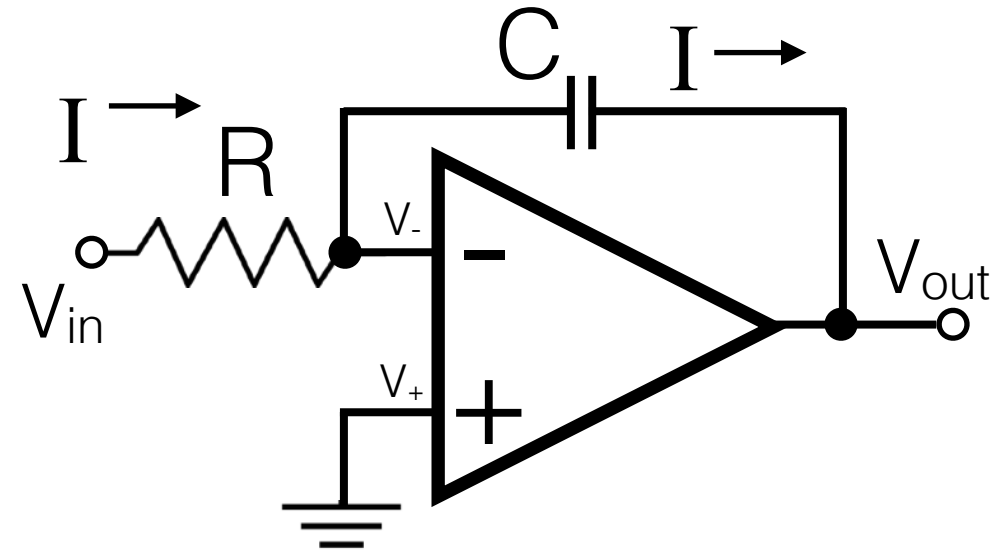
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 $I_R = I_C = I$

$$V_{in} = IR$$

Op-amp integrator

Using the golden rules we can analyze the circuit for an integrator

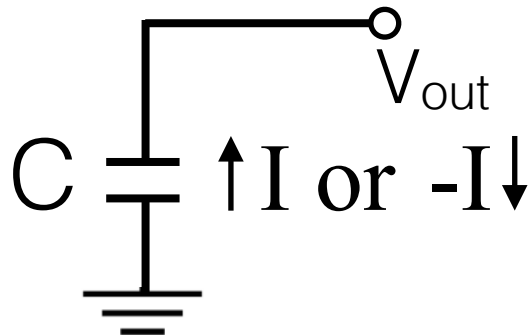


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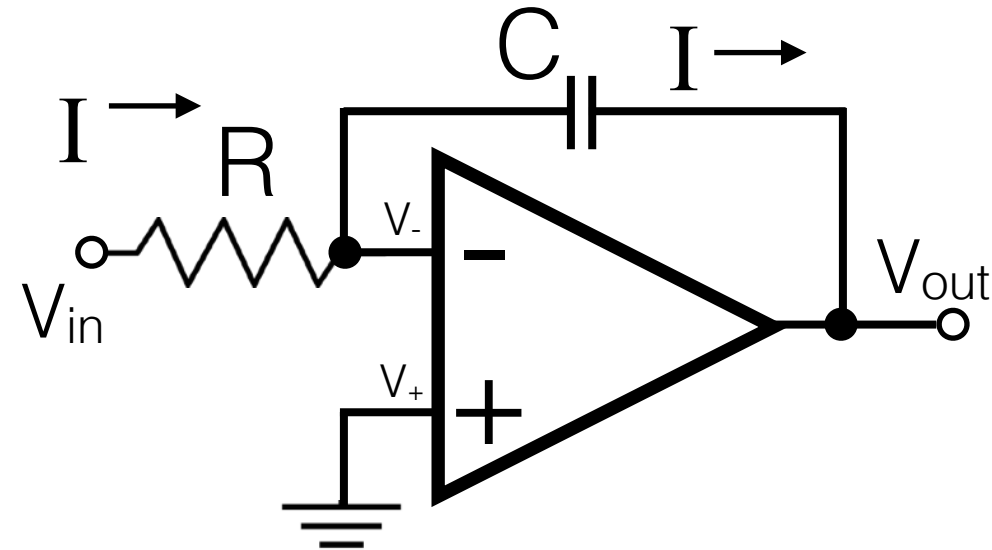
$$V_{in} = IR$$

$$dV_{out}/dt = -I/C = -V_{in}/RC$$



Op-amp integrator

Using the golden rules we can analyze the circuit for an integrator



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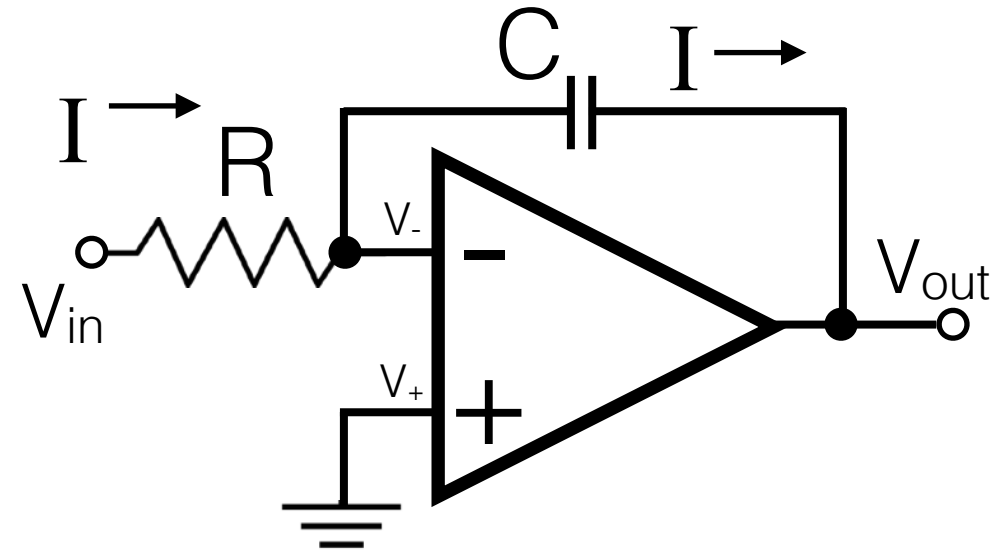
$$V_{in} = I R$$

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$$dV_{out}(t) = -V_{in}(t) dt / RC$$

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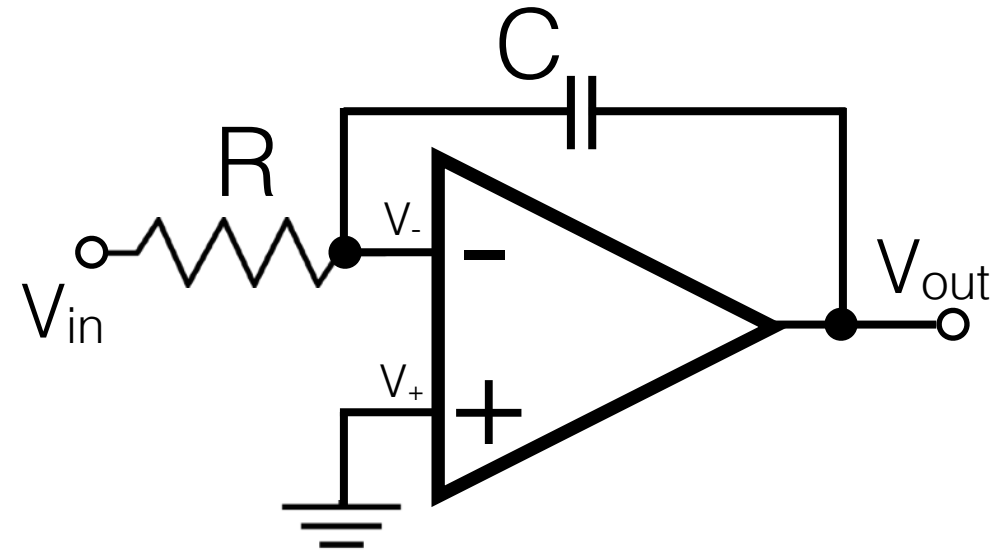
$$\int dV_{out} = -(1/RC) \int V_{in} dt$$

$$V_{out}(t) = -(1/RC) \int V_{in}(t) dt$$

So we get the integrator behavior we saw before, but *without approximation*.

Op-amp integrator

Using the golden rules we can analyze the circuit for an integrator

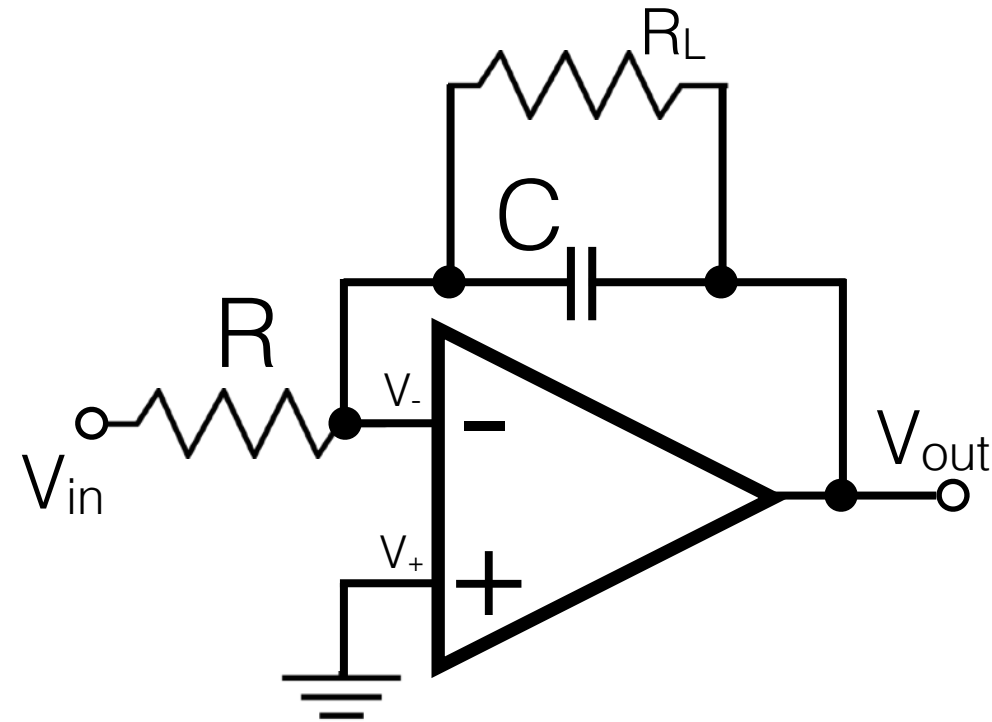


This will integrate forever, and a small input offset will eventually cause it to saturate.

$$V_{out}(t) = -(1/RC) \int V_{in}(t) dt$$

Op-amp integrator

Using the golden rules we can analyze the circuit for an integrator



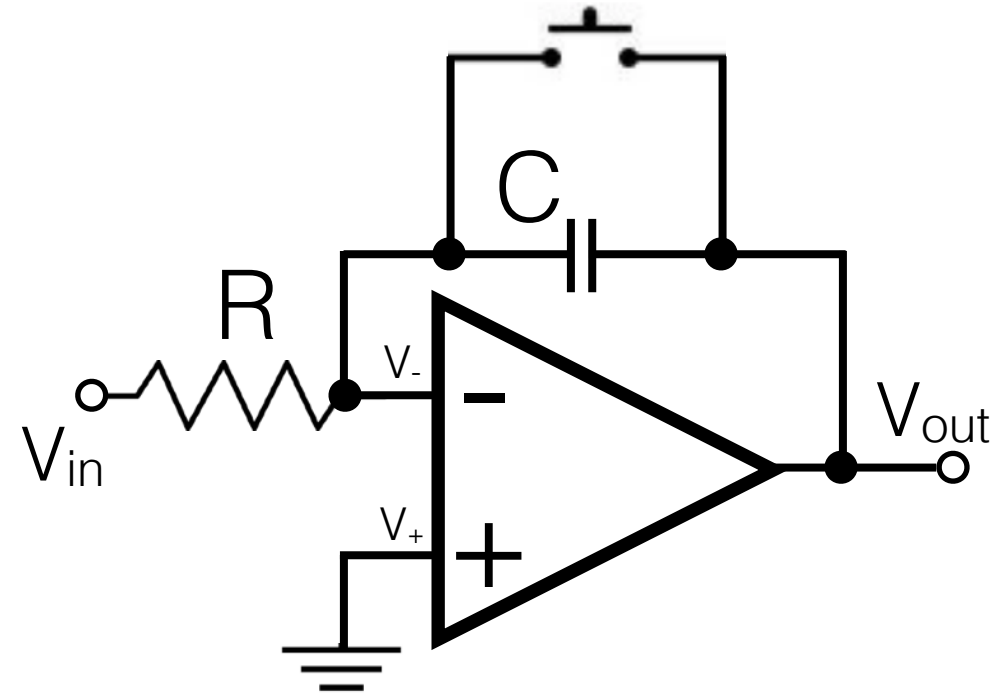
This will integrate forever, and a small input offset will eventually cause it to saturate.

Can leak off that \sim DC build up with a large "leakage" resistor.

$$V_{out}(t) = -(1/RC) \int V_{in}(t) dt$$

Op-amp integrator

Using the golden rules we can analyze the circuit for an integrator

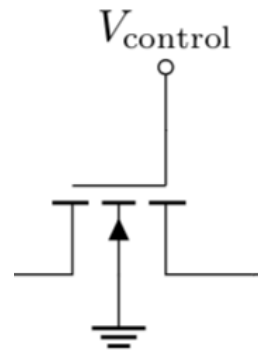


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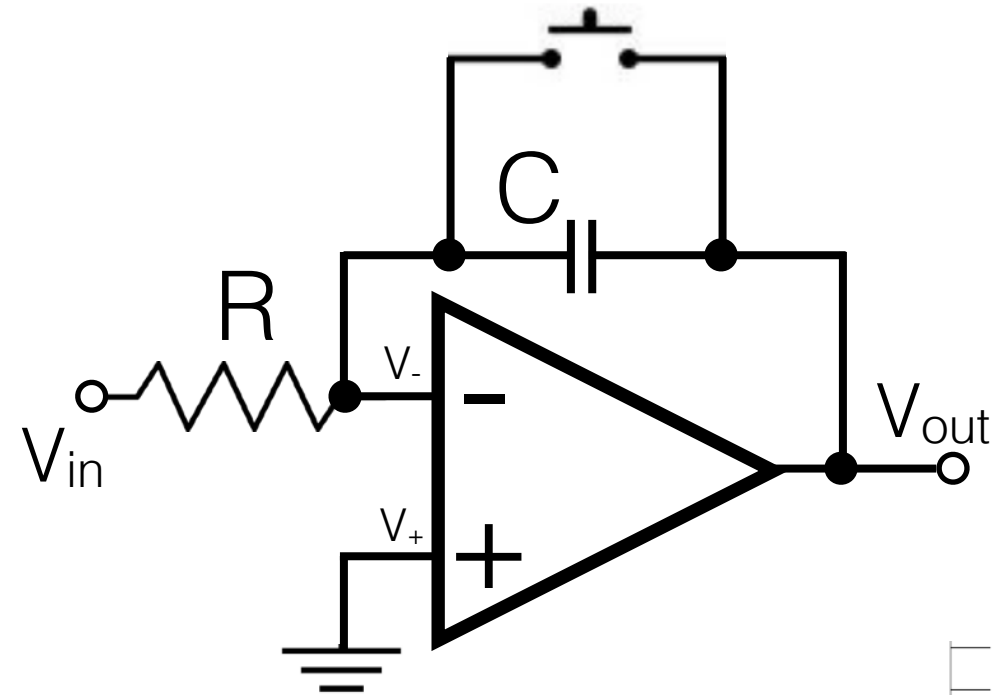
Can leak off that ~DC build up with a large resistor.

Or specifically reset it at appropriate times with a switch.
Can use a FET switch for computer control.

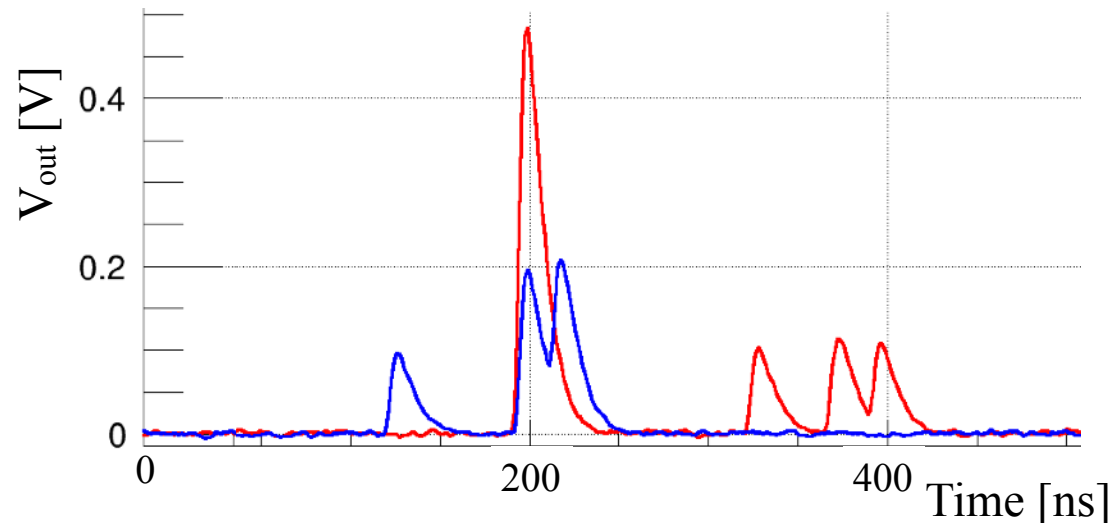


Op-amp integrator

Using the golden rules we can analyze the circuit for an integrator

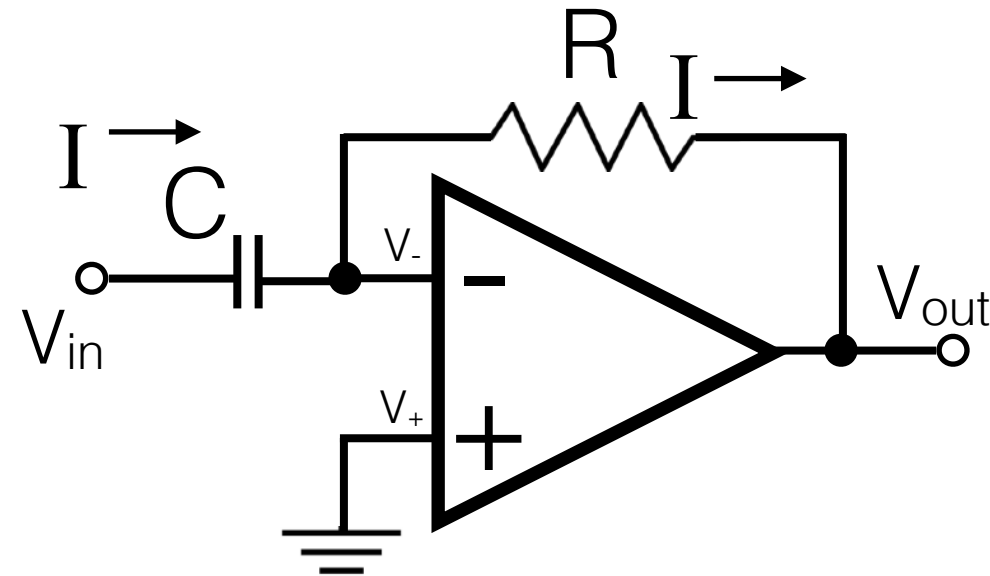


$$V_{out}(t) = -(1/RC) \int V_{in}(t) dt$$



Op-amp differentiator

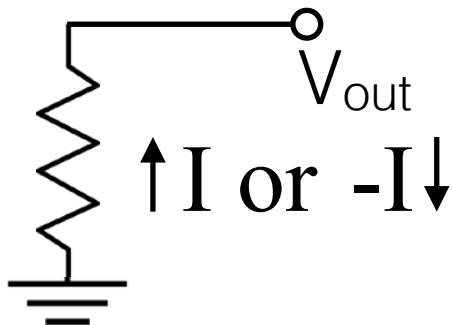
Using the golden rules we can analyze the circuit for a differentiator



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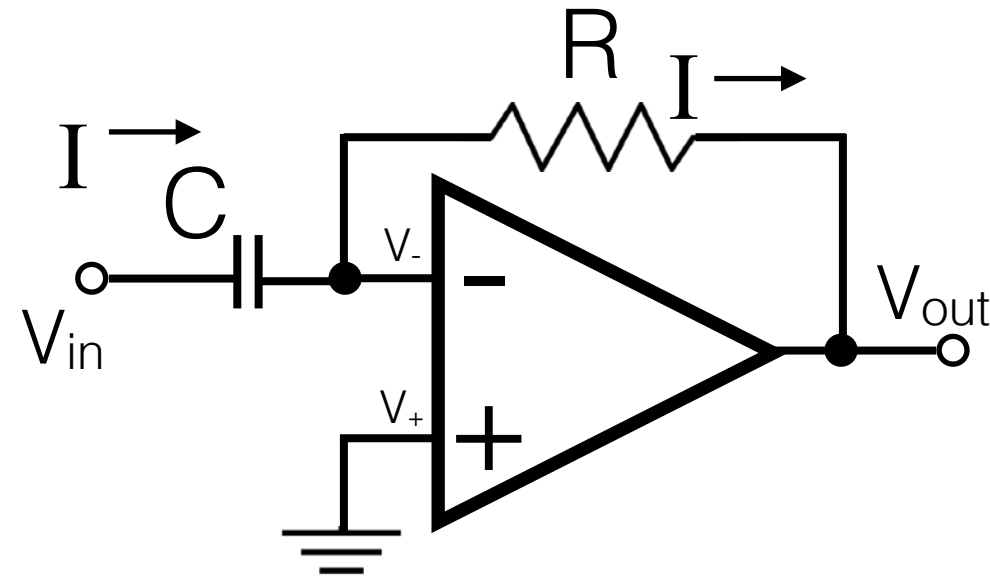
Rule 1 means that no current flows into inverting input. So,
 $I_R = I_C = I$

$$V_{out} = -IR$$



Op-amp differentiator

Using the golden rules we can analyze the circuit for a differentiator



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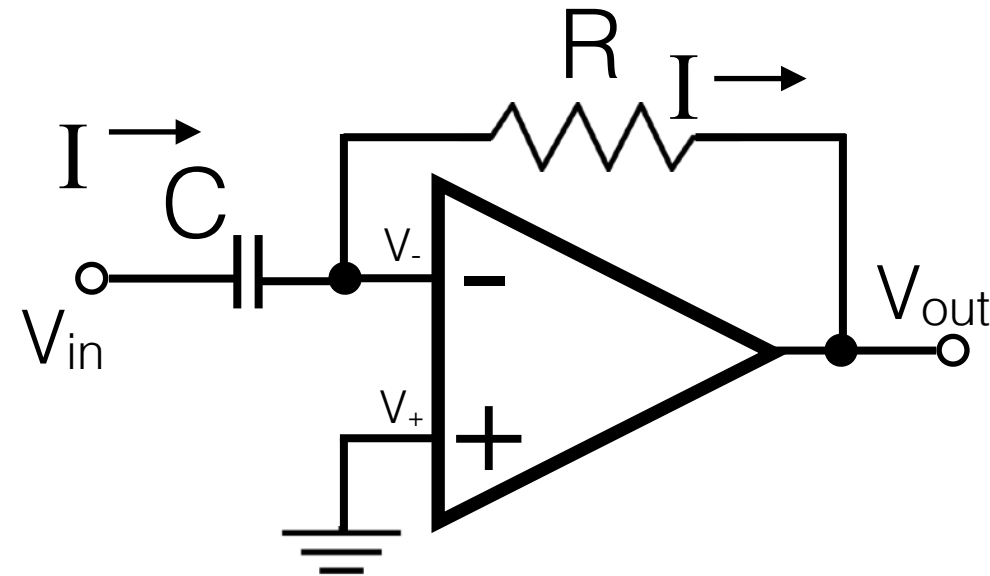
$$V_{out} = -IR \rightarrow I = -V_{out} / R$$

$$dV_{in}/dt = I / C = -V_{out} / RC$$

$$V_{out} = -RC dV_{in}/dt$$

Op-amp differentiator

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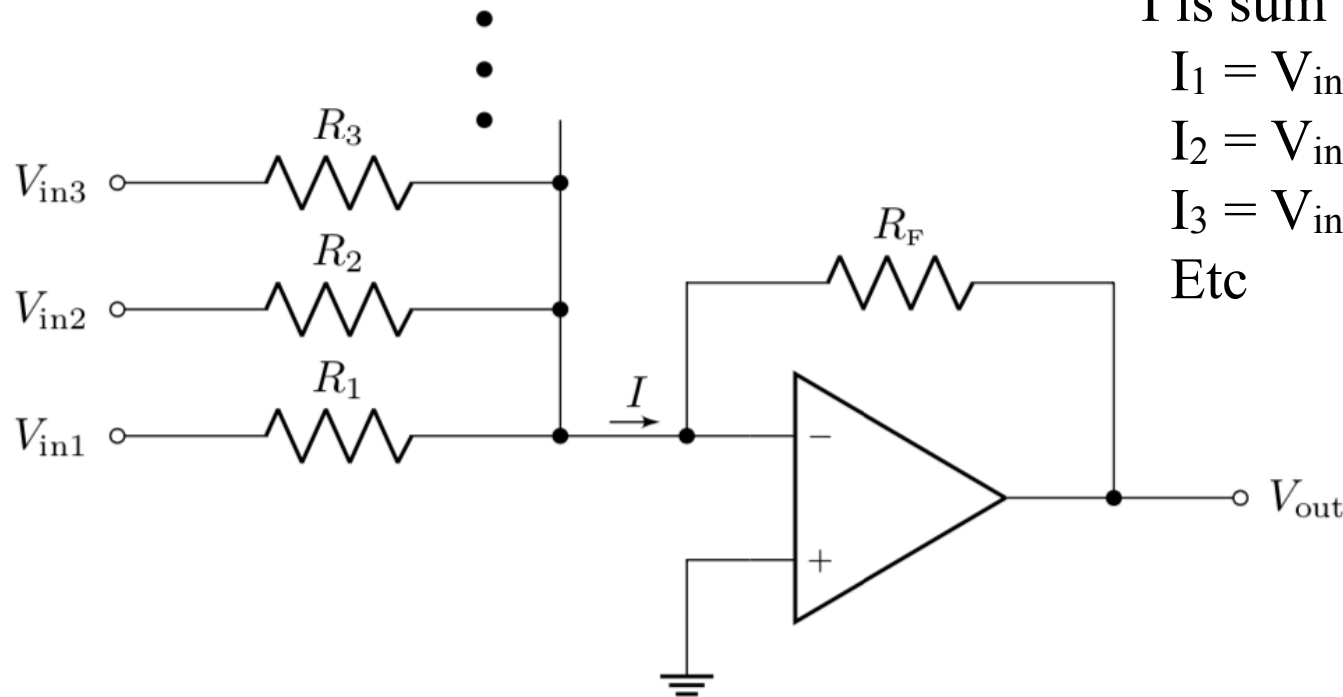
$$dV_{in}/dt = I / C = -V_{out} / RC$$

$$V_{out} = -RC dV_{in}/dt$$

$$V_{out}(t) = -RC dV_{in}(t)/dt$$

Summing amplifier

Using the golden rules we can analyze the circuit for a summing amp



$$V_{out} = - I R_F$$

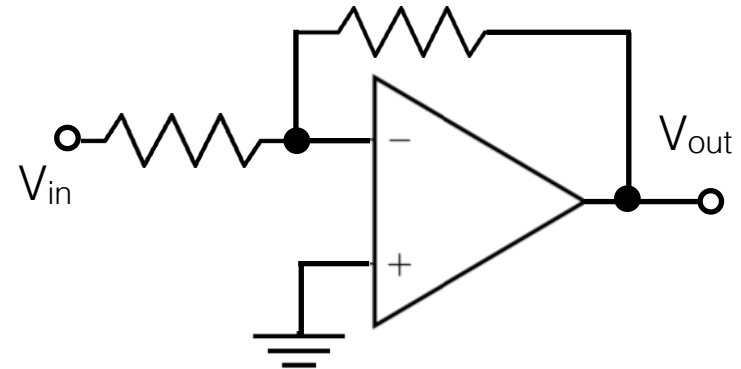
I is sum of all input currents

$$I_1 = V_{in1}/R_1$$

$$I_2 = V_{in2}/R_2$$

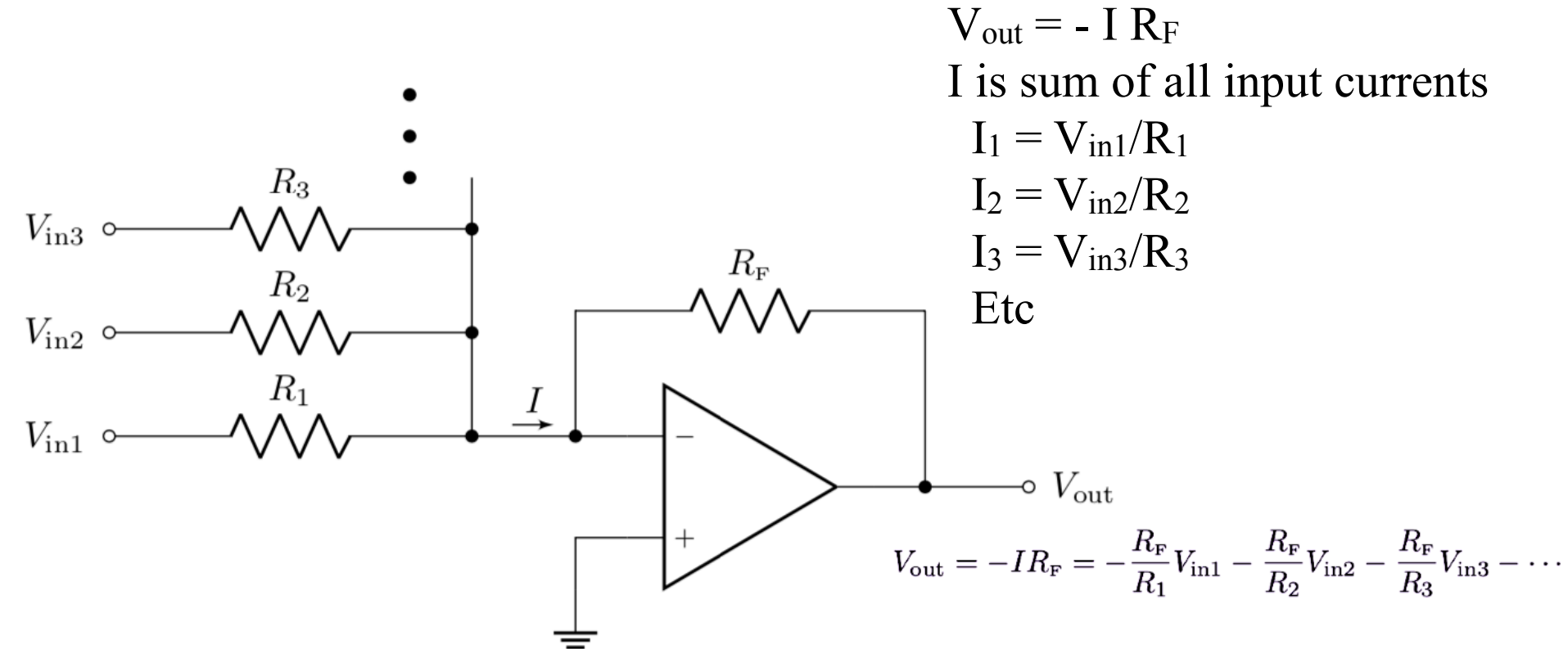
$$I_3 = V_{in3}/R_3$$

Etc



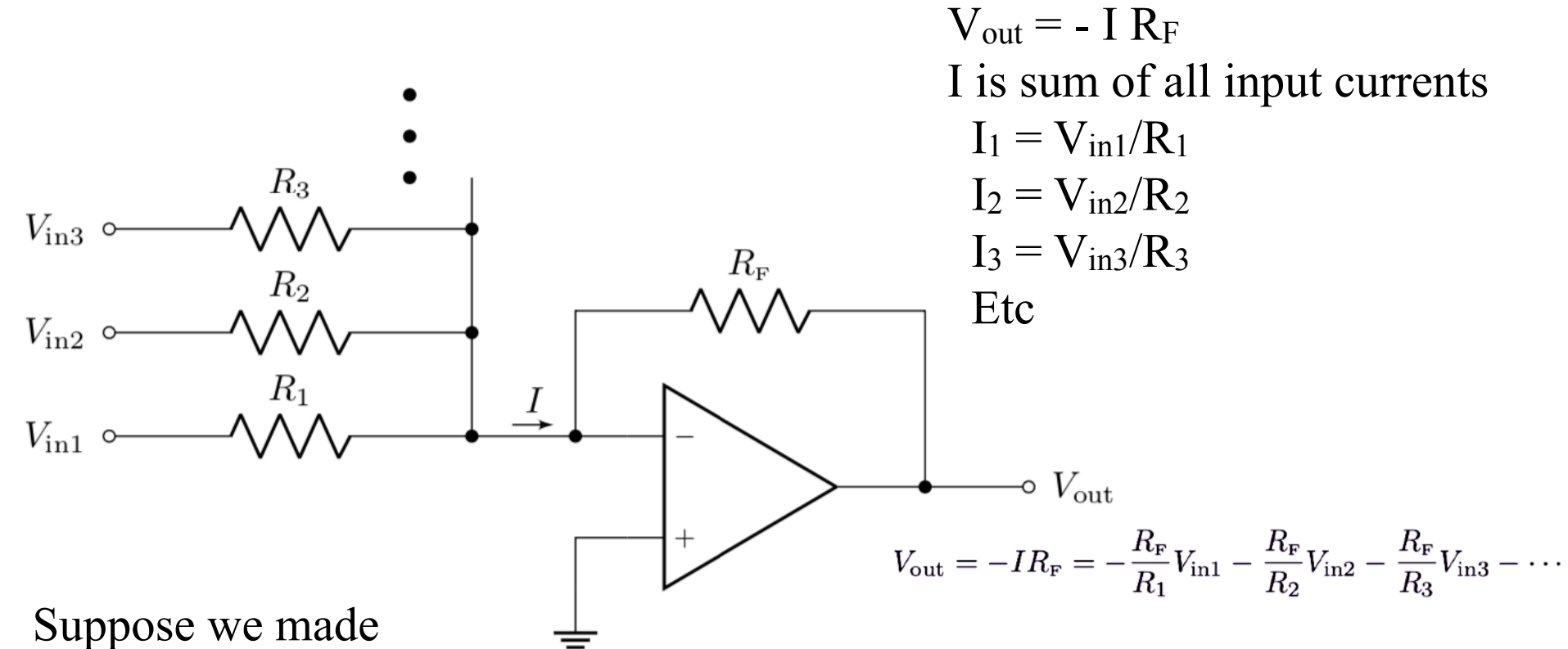
Summing amplifier

Using the golden rules we can analyze the circuit for a summing amp



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Suppose we made

$$R_1 = R_F$$

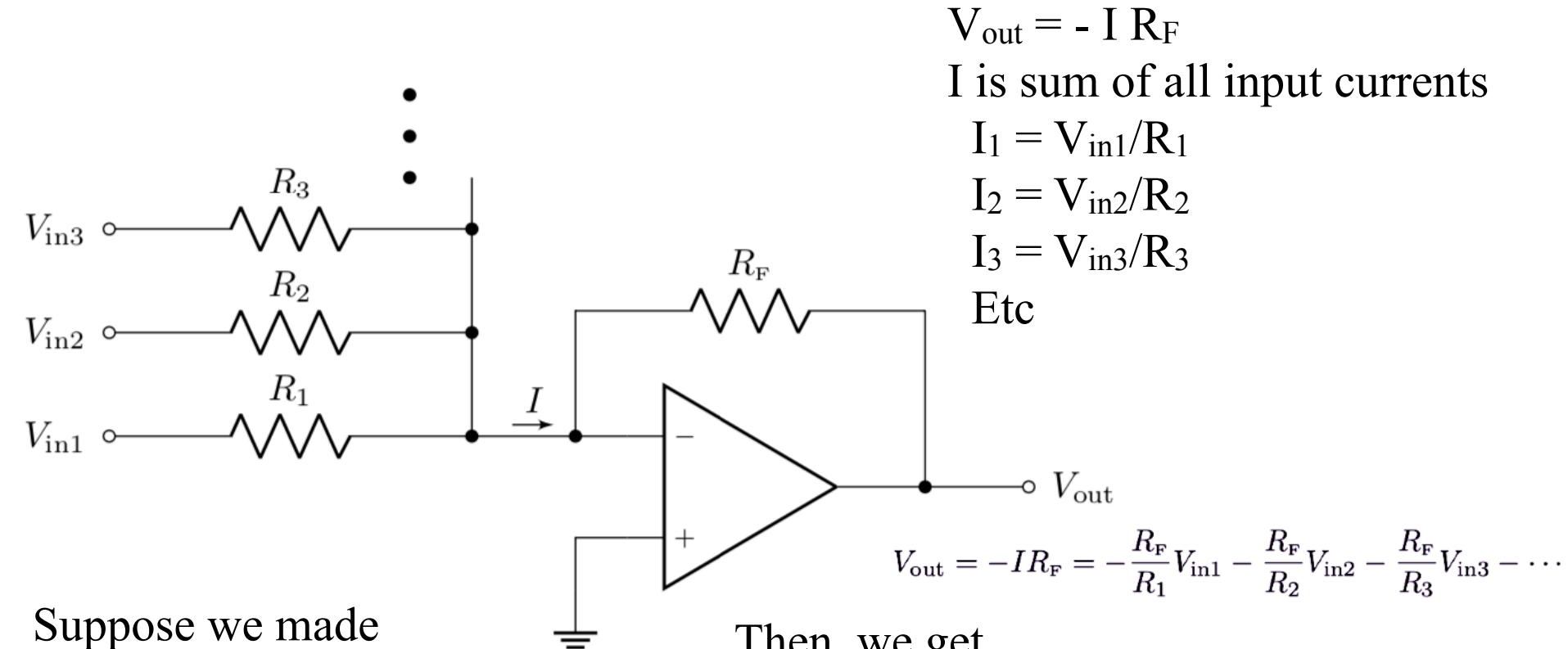
$$R_2 = R_F/2$$

$$R_3 = R_F/4$$

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Summing amplifier

Using the golden rules we can analyze the circuit for a summing amp



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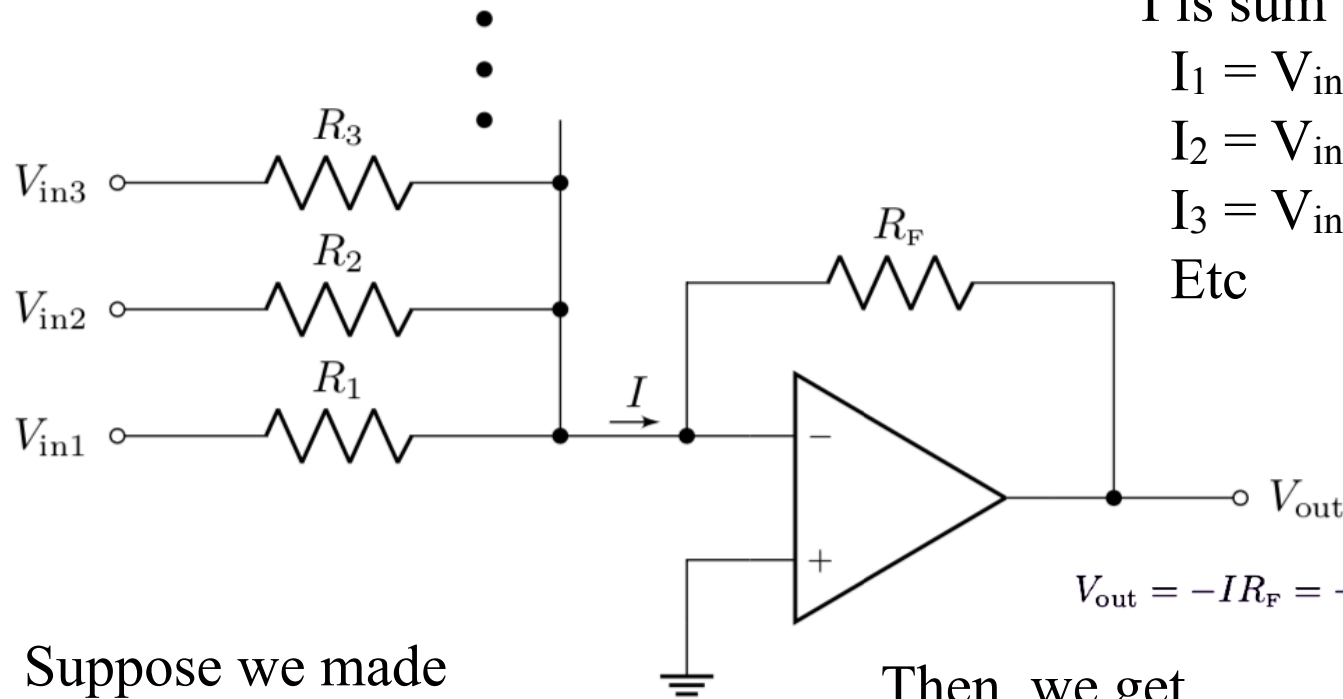
Then, we get

$$V_{\text{out}} = -V_{\text{in}1} - 2V_{\text{in}2} - 4V_{\text{in}3} - 8V_{\text{in}4}$$

This allows the inputs to be binary-coded signals, e.g., 0.0 and 0.1 V, then the output reconstructs the analog voltage.

Summing amplifier

Using the golden rules we can analyze the circuit for a summing amp



$$V_{\text{out}} = - I R_F$$

I is sum of all input currents

$$I_1 = V_{\text{in}1}/R_1$$

$$I_2 = V_{\text{in}2}/R_2$$

$$I_3 = V_{\text{in}3}/R_3$$

Etc

$$V_{\text{out}} = -IR_F = -\frac{R_F}{R_1}V_{\text{in}1} - \frac{R_F}{R_2}V_{\text{in}2} - \frac{R_F}{R_3}V_{\text{in}3} - \dots$$

Suppose we made

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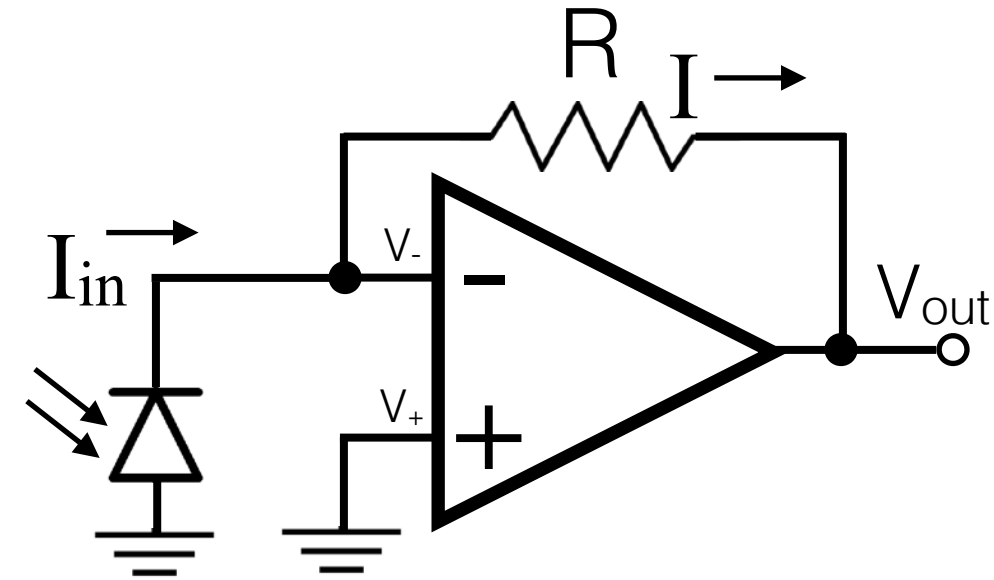


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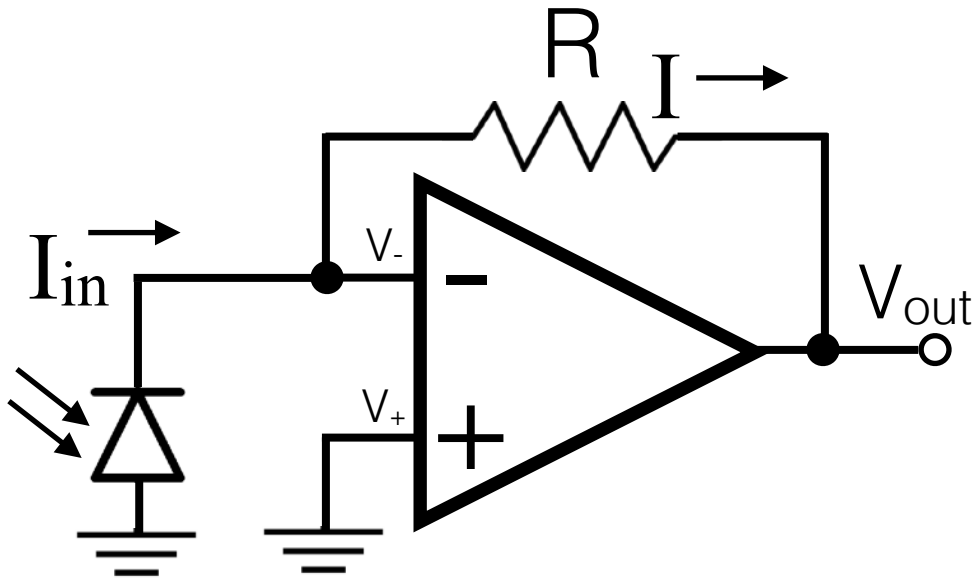
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Op-amp current amplifier

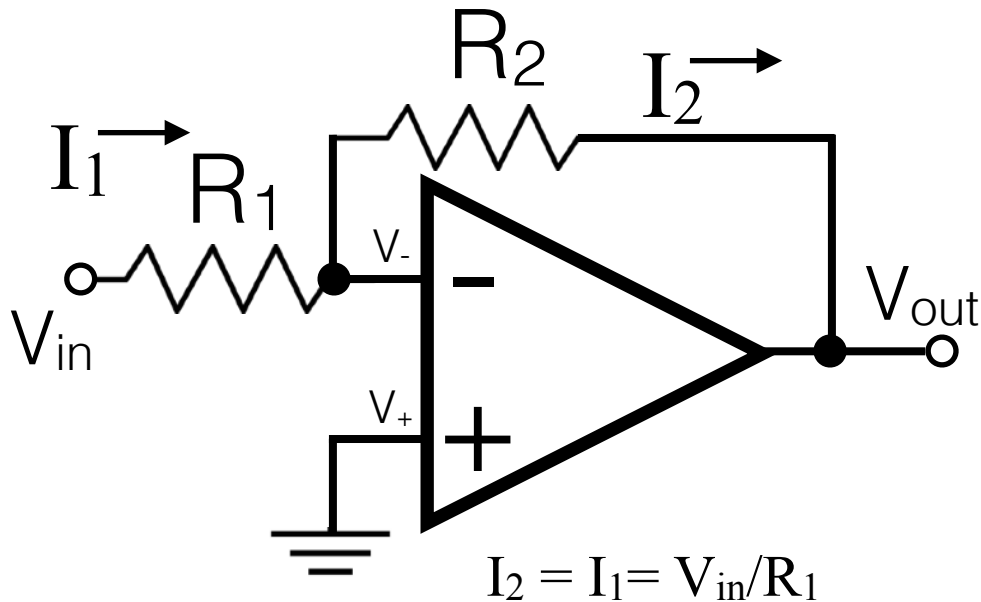


Op-amp current amplifier



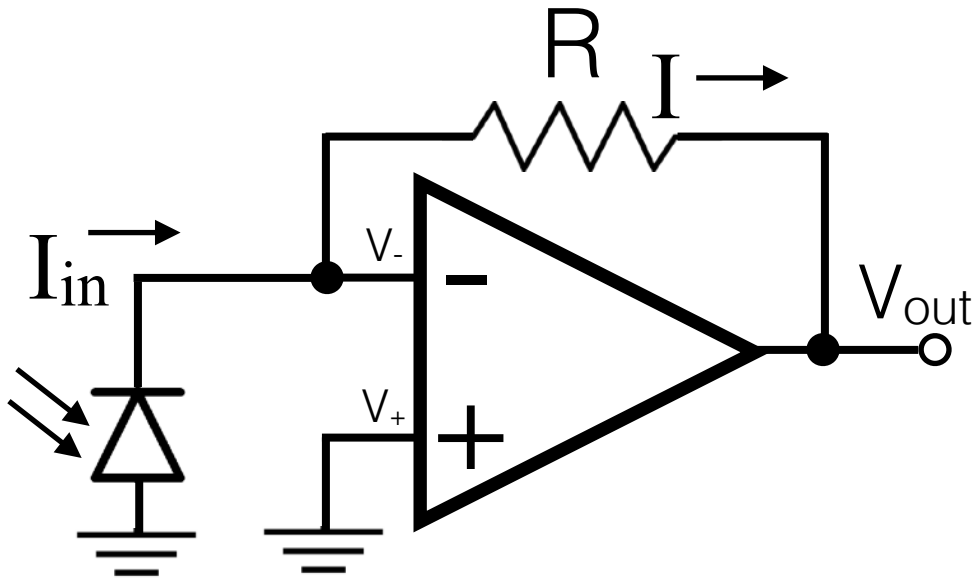
Rule 2 means $V_- = \text{ground}$.
Called a “virtual ground”.

Rule 1 means that no current flows into inverting input. So,
 $I_R = I_{in}$



$$I_2 = I_1 = V_{in}/R_1$$

Op-amp current amplifier



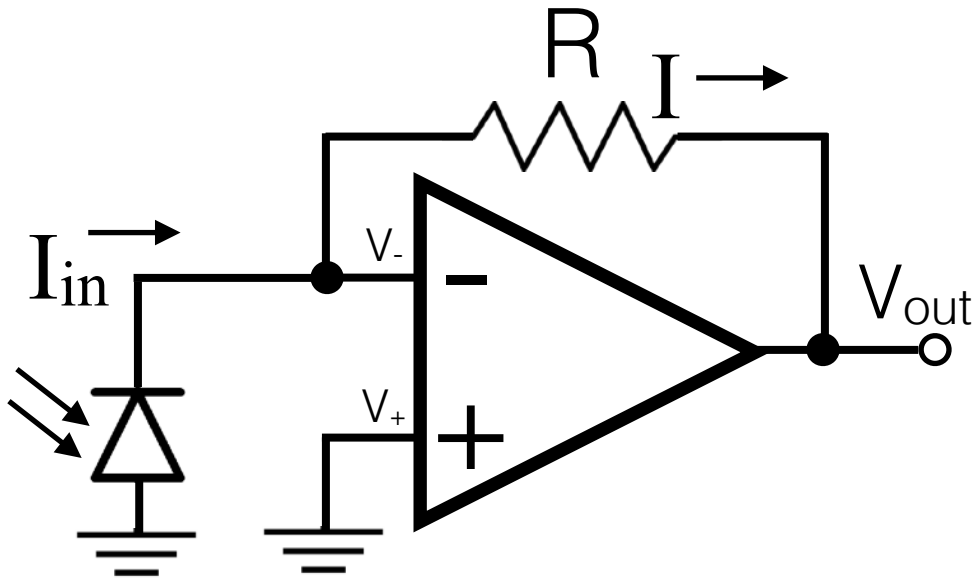
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If $R = 1\text{M}\Omega$ then we get $1\text{ V}/\mu\text{A}$.

Op-amp current amplifier

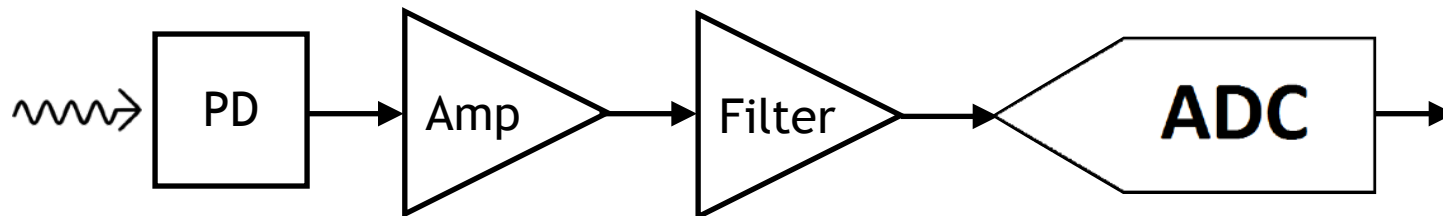


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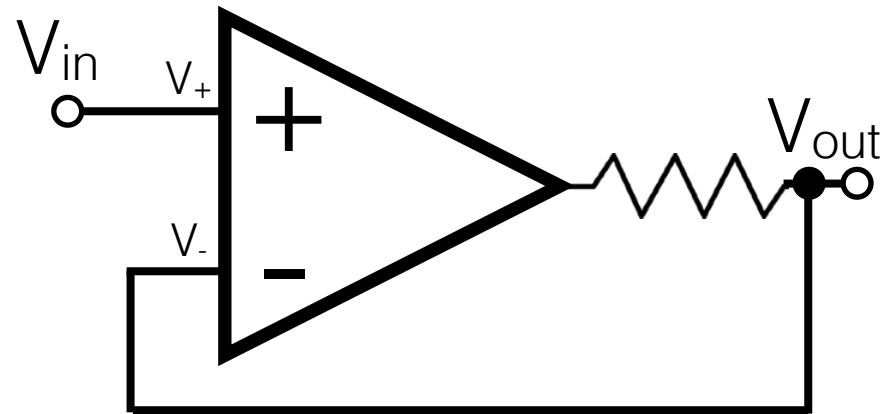
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More complex feedback loops

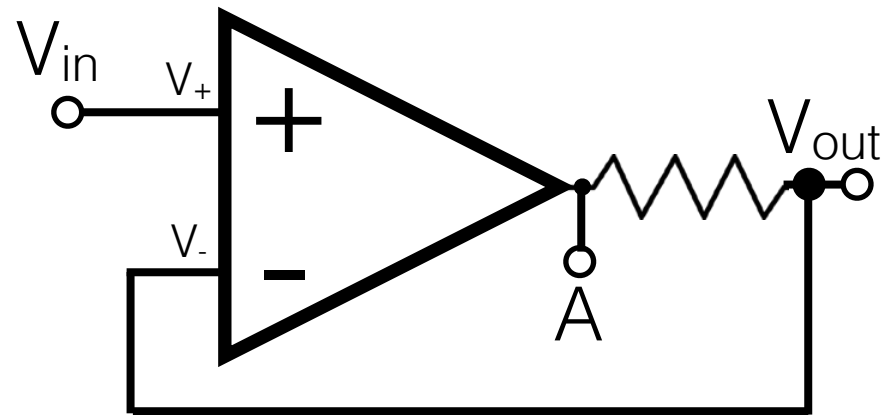
The golden rules work even when we have other things in the feedback loop



Rule 2 makes $V_{out} = V_{in}$ regardless of any voltage dropped across the resistor.

More complex feedback loops

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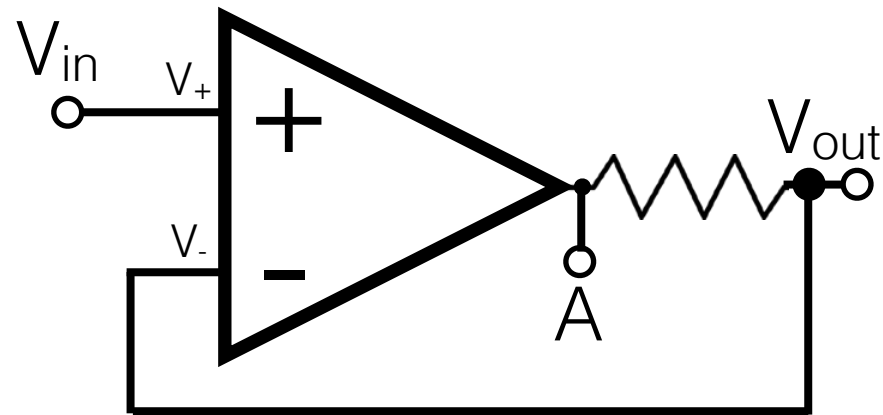


Rule 2 makes $V_{out} = V_{in}$ regardless of any voltage dropped across the resistor.

The op-amp output, A, will have to be higher than V_{out} to make $V_{out} = V_{in}$, but the op-amp can do that, with a tiny difference between V_+ and V_- .

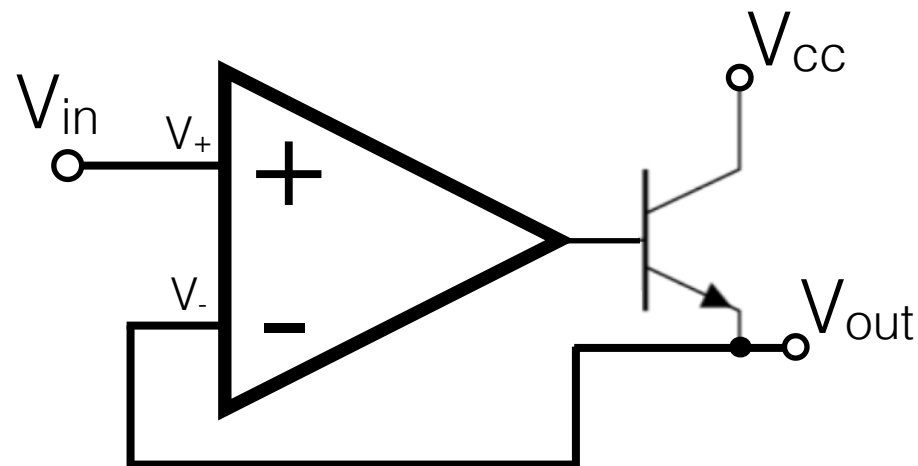
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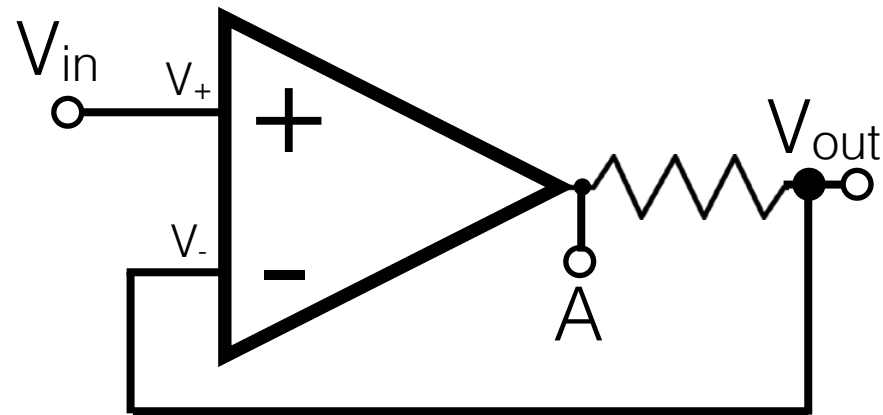
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What about a transistor in the loop?

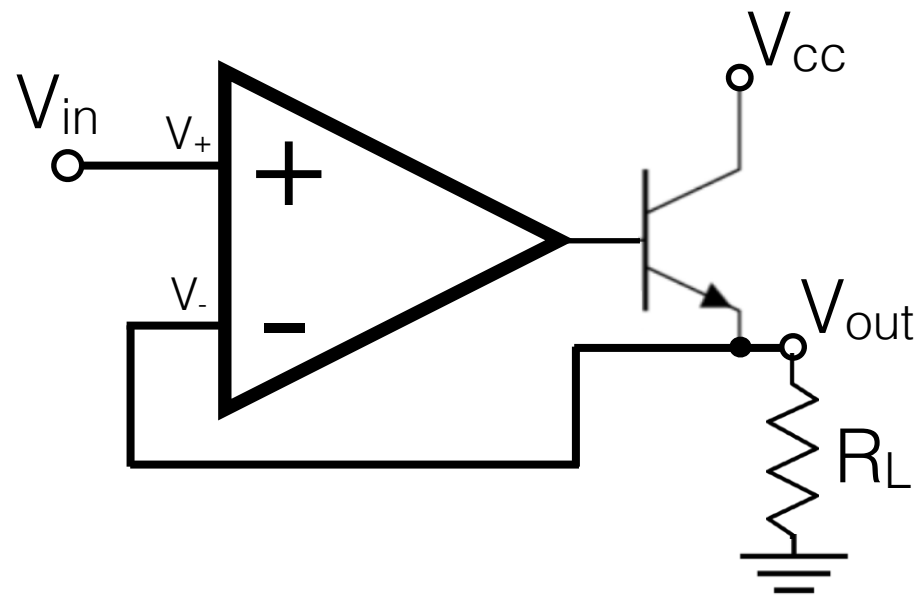
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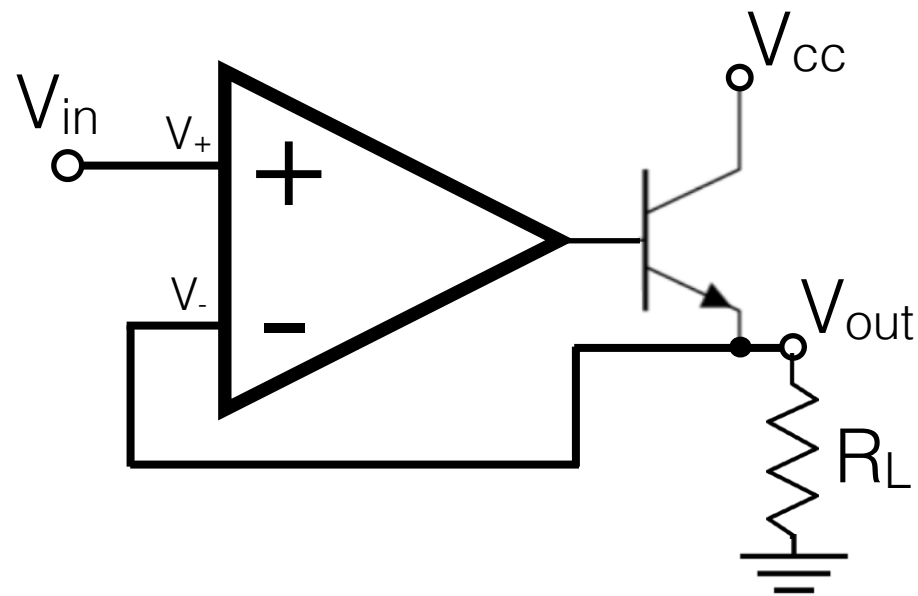
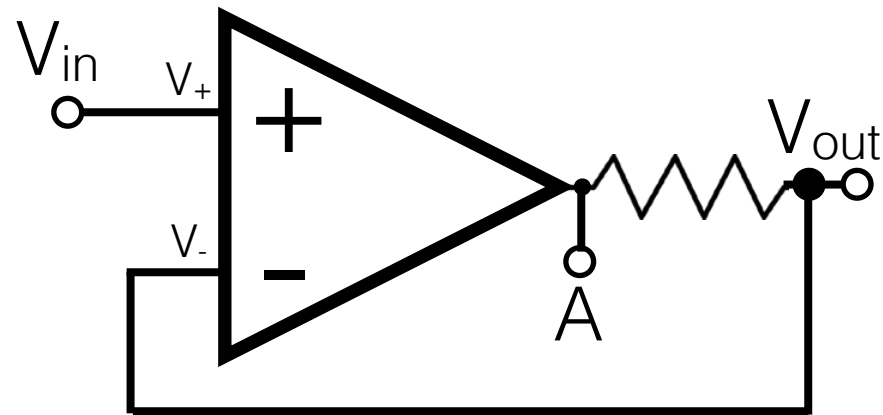
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What about a transistor in the loop?
The load can be the emitter resistor, which can be a low resistance, e.g., a speaker, with a high power transistor driving it.

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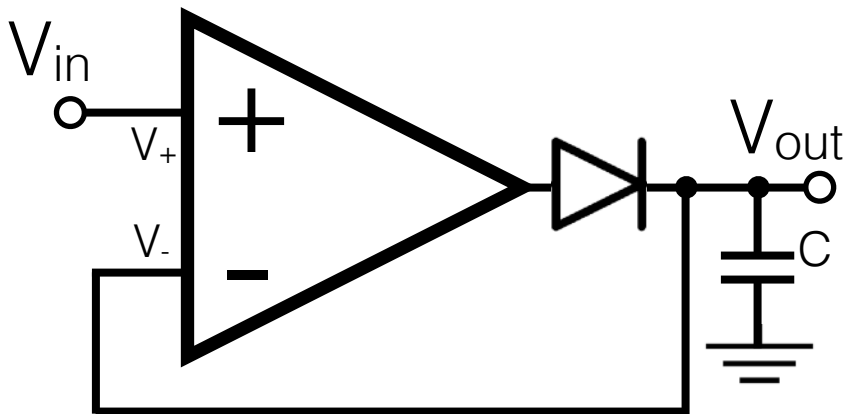
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What about a transistor in the loop? The load can be the emitter resistor, which can be a low resistance, e.g., a speaker, with a high power transistor driving it.

The 0.6 V diode drop is *compensated* by the op-amp, so $V_{out} = V_{in}$ without a 0.6 V drop.

Peak detector

The golden rules work even when we have other things in the feedback loop



The output will follow on the way up.

The op-amp will compensate the diode drop.

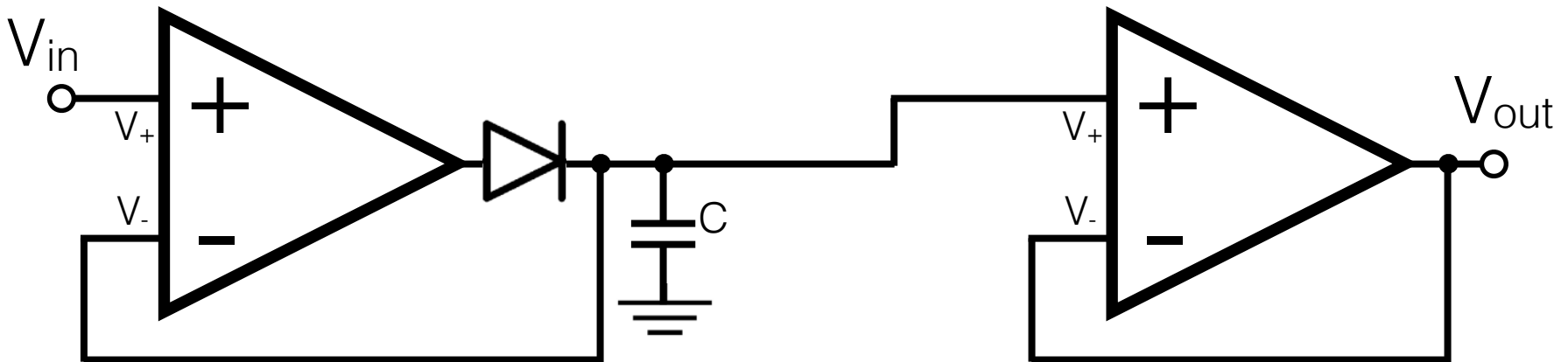
The diode won't let the op-amp pull it back down.

The capacitor will hold the maximum voltage reached.

The V_- input will not allow current flow to discharge the capacitor.

Peak detector

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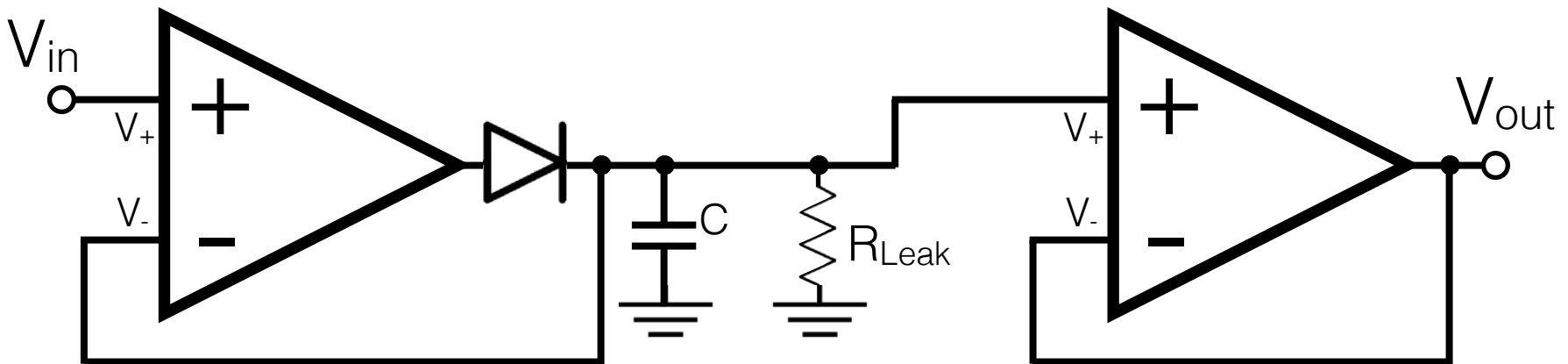
The capacitor will hold the maximum voltage reached.

The V_- input will not allow current flow to discharge the capacitor.

Can add a follower to avoid discharging the capacitor through the load.

Peak detector

The golden rules work even when we have other things in the feedback loop



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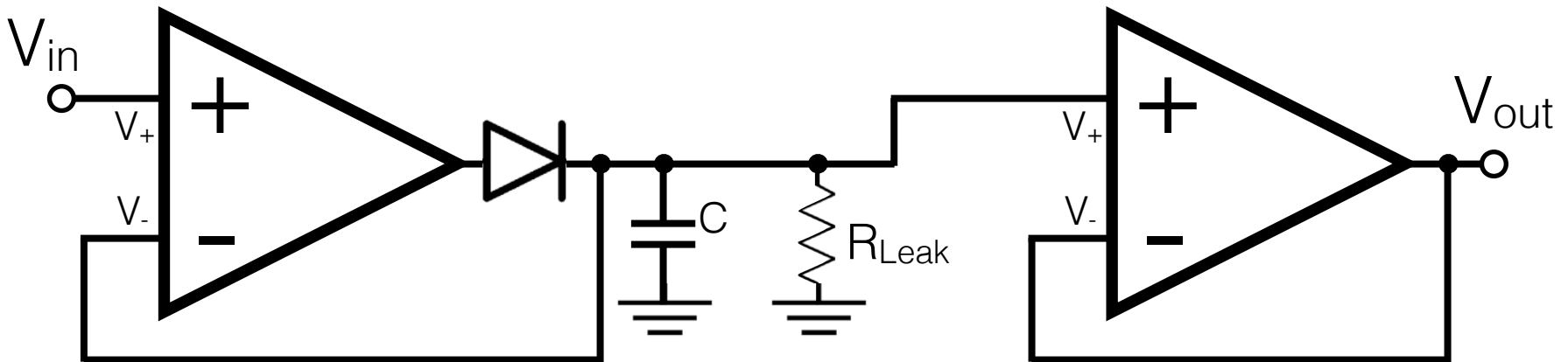
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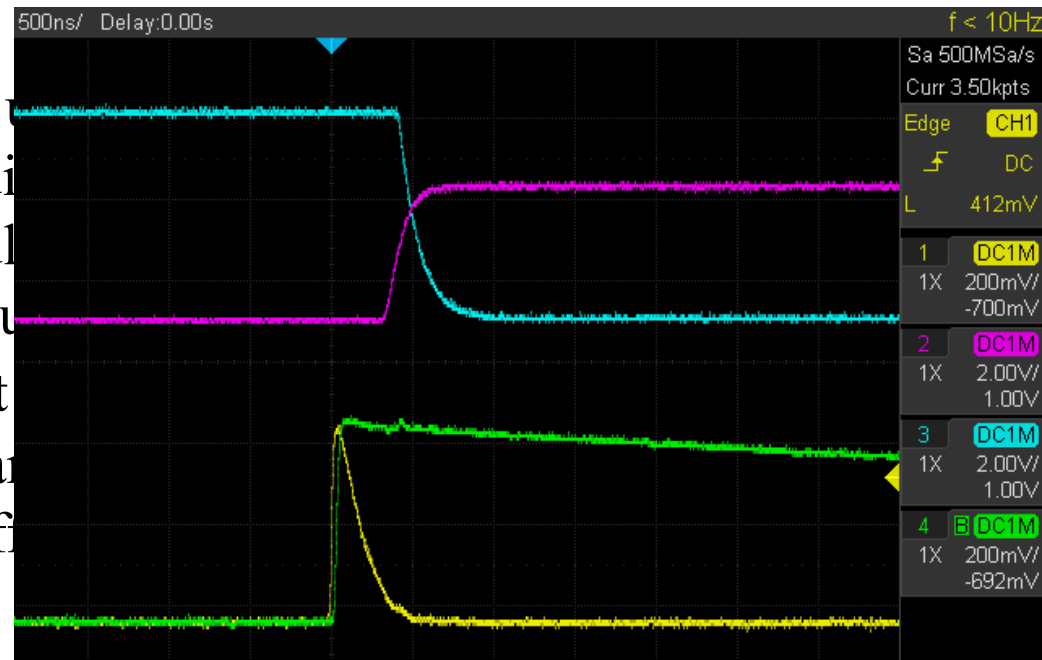
Add a controlled resistor to leak off the charge with a long time constant.

Peak detector

The golden rules work even when we have other things in the feedback loop

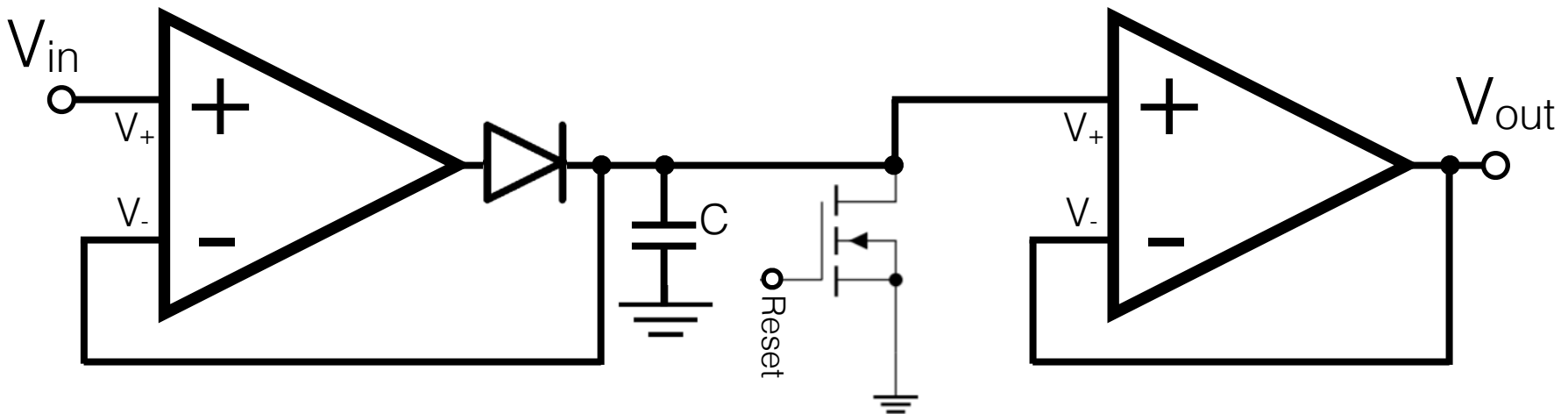


The output will follow on the way up
The op-amp will compensate the diode drop
The diode won't let the op-amp pull the output down
The capacitor will hold the maximum value
The V_- input will not allow current to flow out
Can add a follower to avoid discharging
Add a controlled resistor to leak off the charge



Peak detector

The golden rules work even when we have other things in the feedback loop



The output will follow on the way up.

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The diode won't let the op-amp pull it back down.

The capacitor will hold the maximum voltage reached.

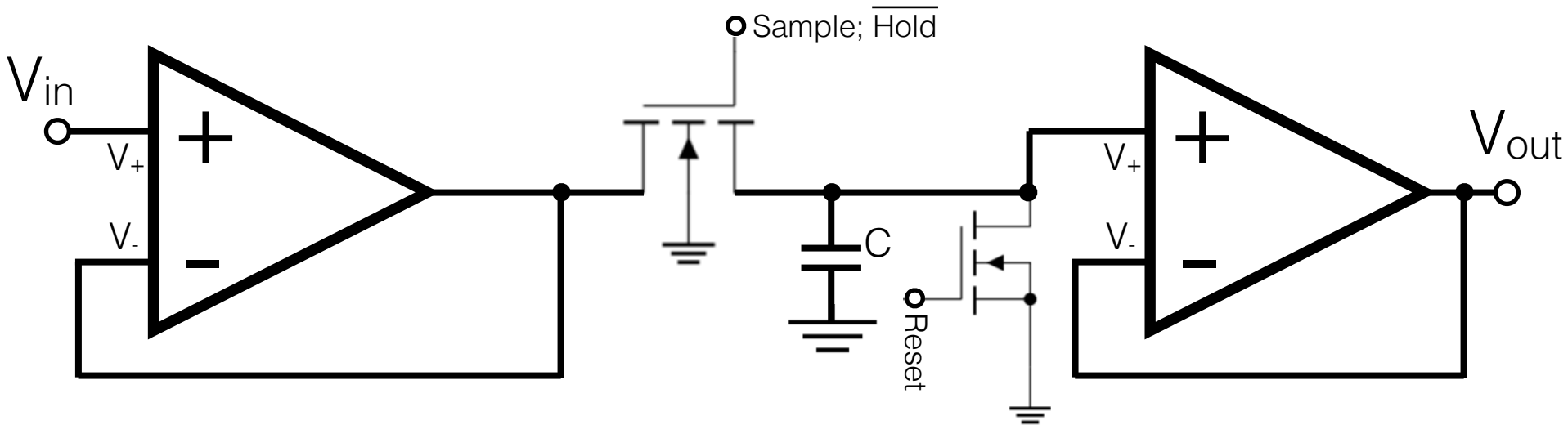
The V_- input will not allow current flow to discharge the capacitor.

Can add a follower to avoid discharging the capacitor through the load.

Add a controlled resistor to leak off the charge with a long time constant.

Or, add a MOSFET switch to reset the capacitor.

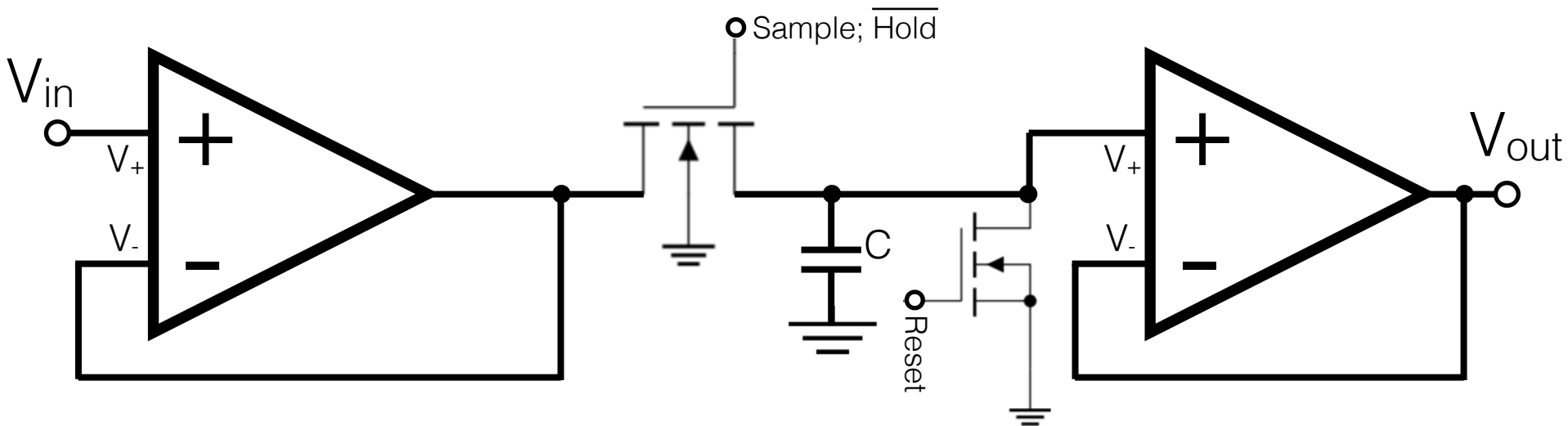
Sample and hold



We can often time measurements, e.g., pump and probe.

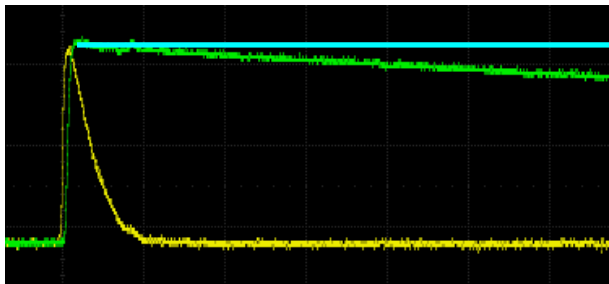
So we can use MOSFET switches from a control computer to time controls:
reset, sample, hold, [readout], reset.

Sample and hold

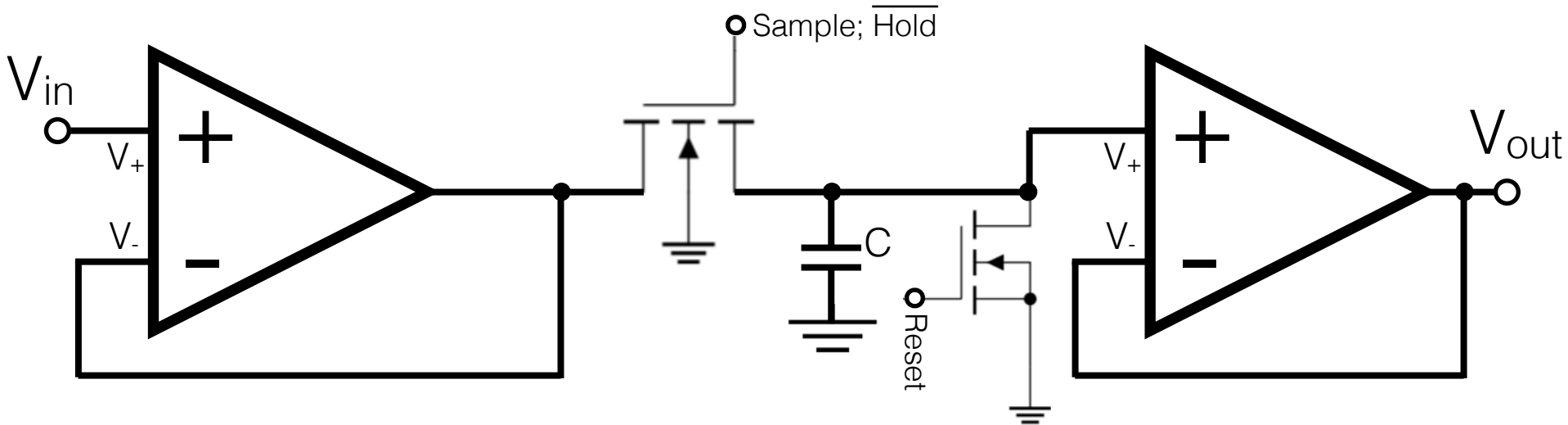


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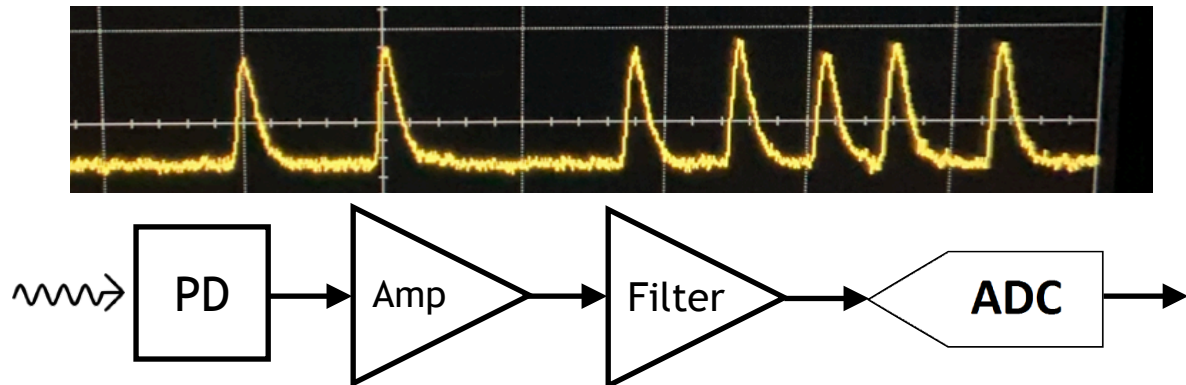
Sample and hold



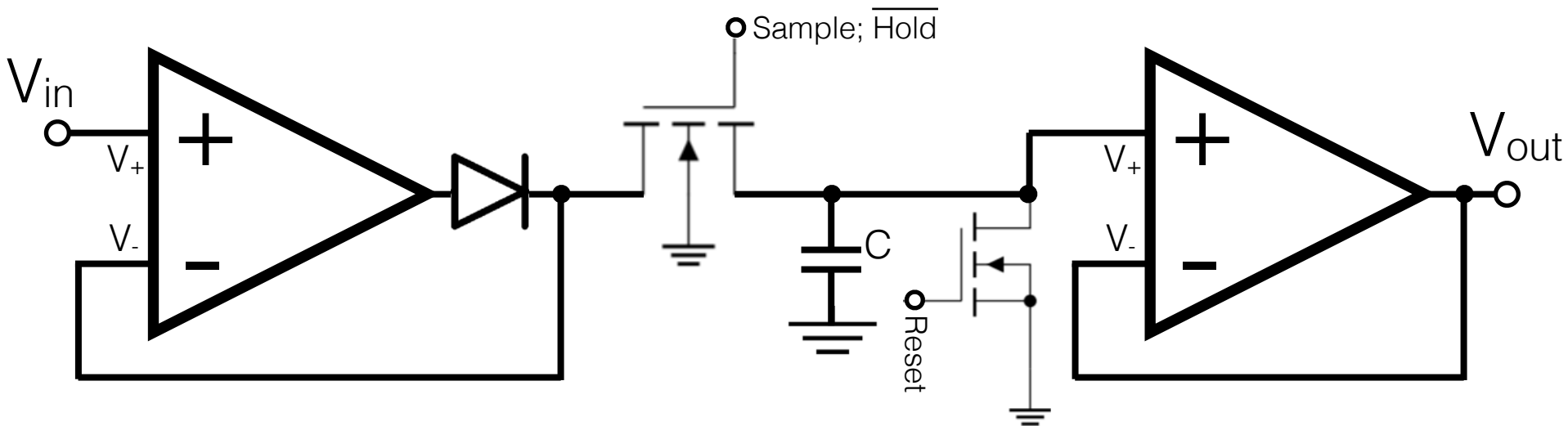
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How could you use something like this to “count” photons per second in our example experiment?



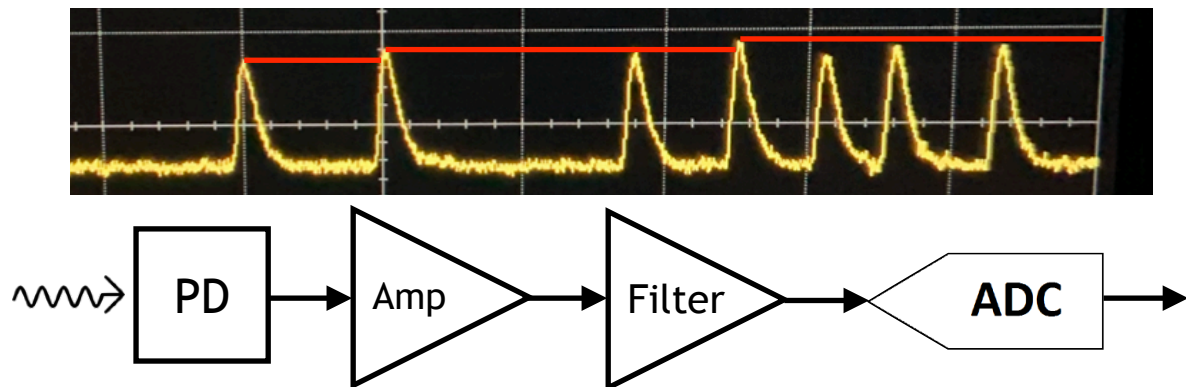
Sample and hold



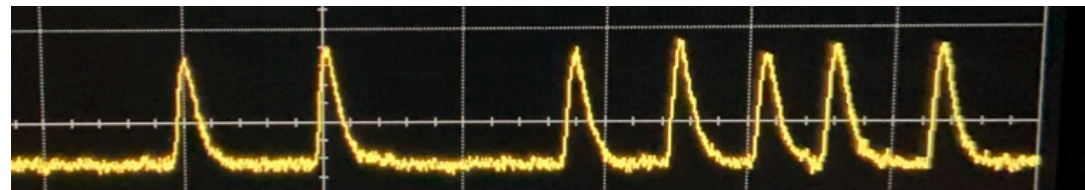
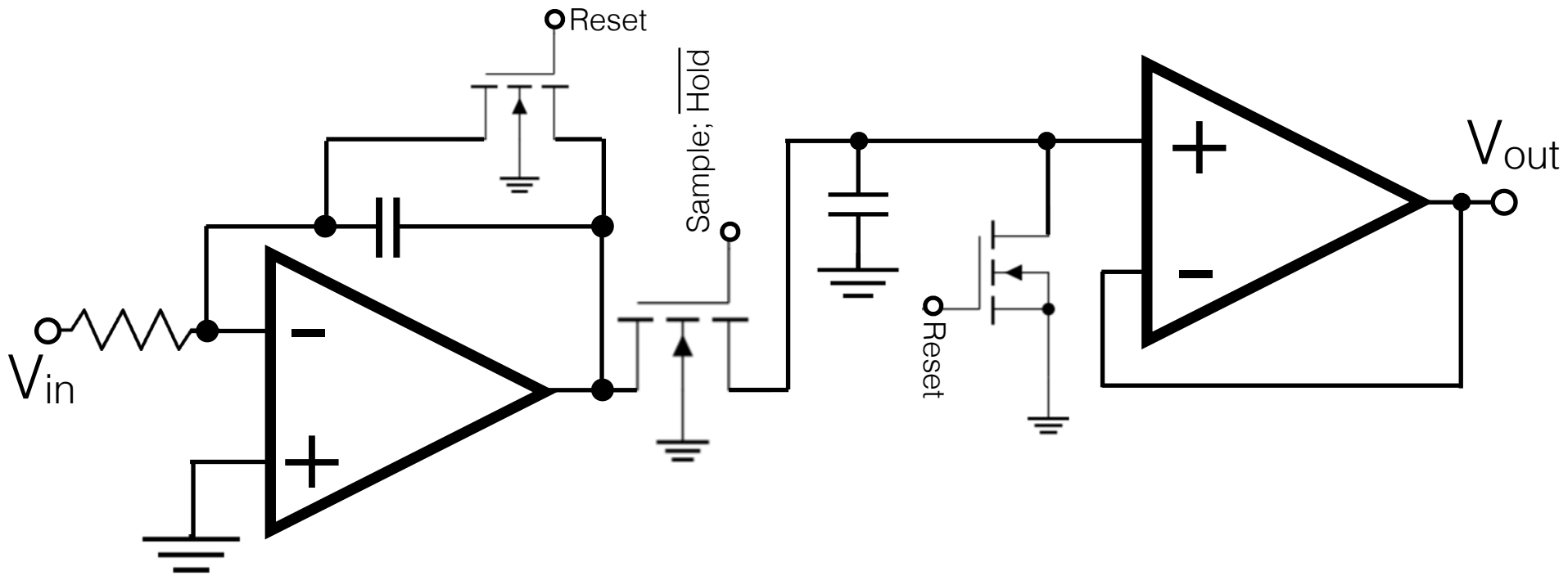
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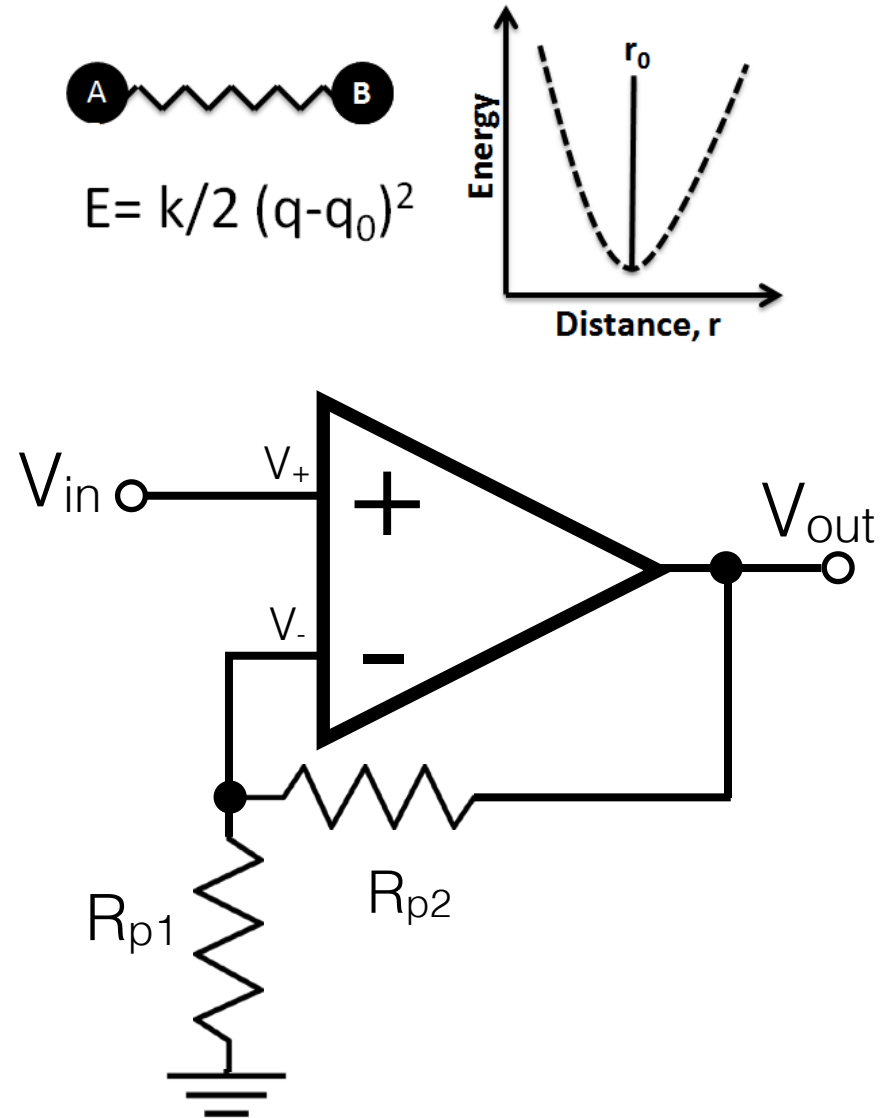
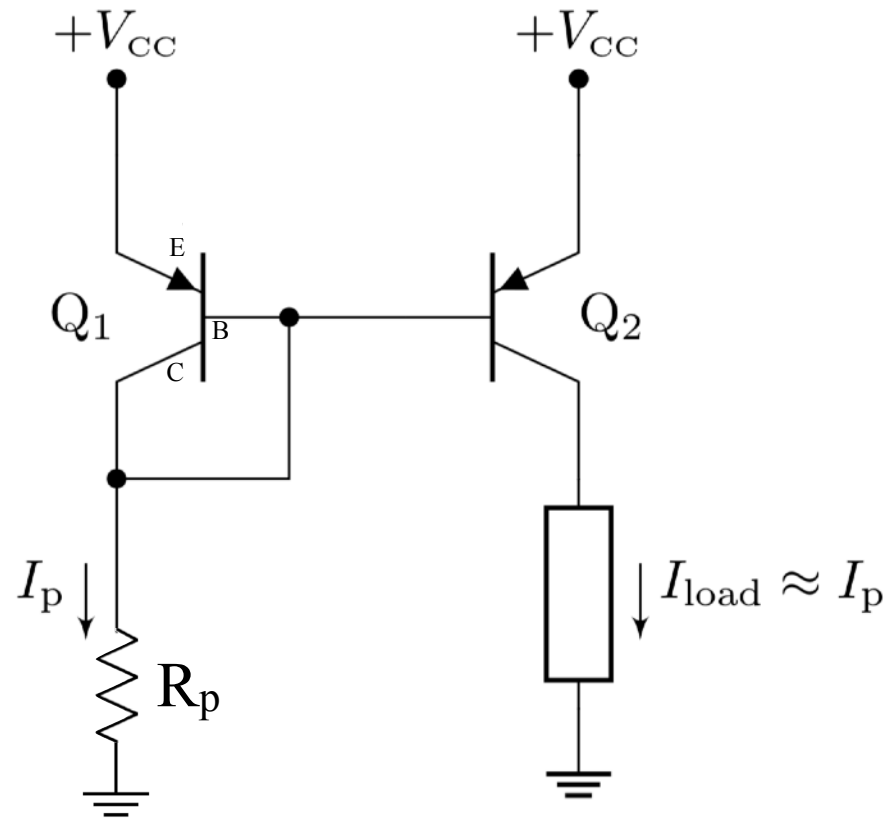


Integrate, sample, and hold



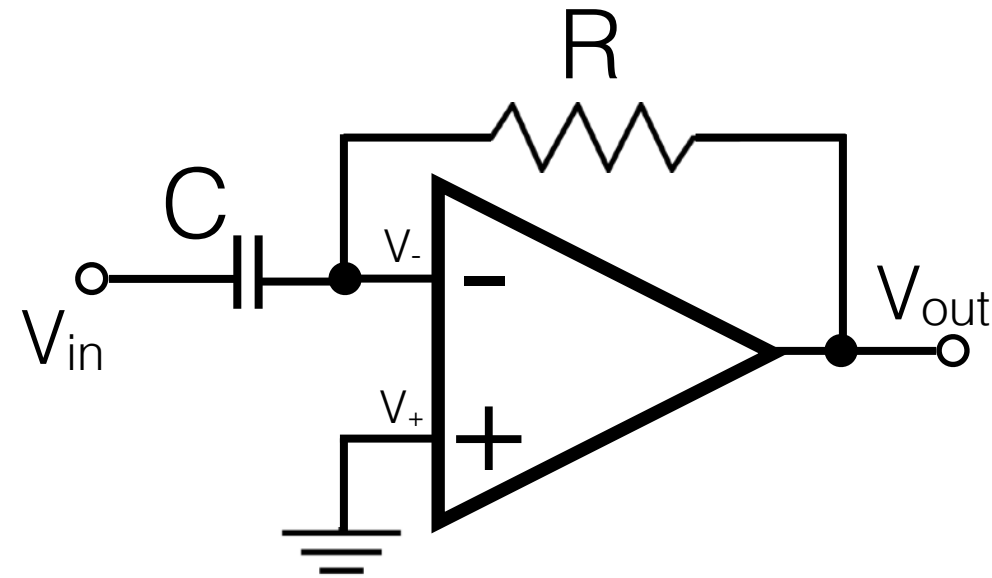
Negative feedback

We also saw negative feedback with a current mirror.



Op-amp differentiator

Using the golden rules we can analyze the circuit for a differentiator

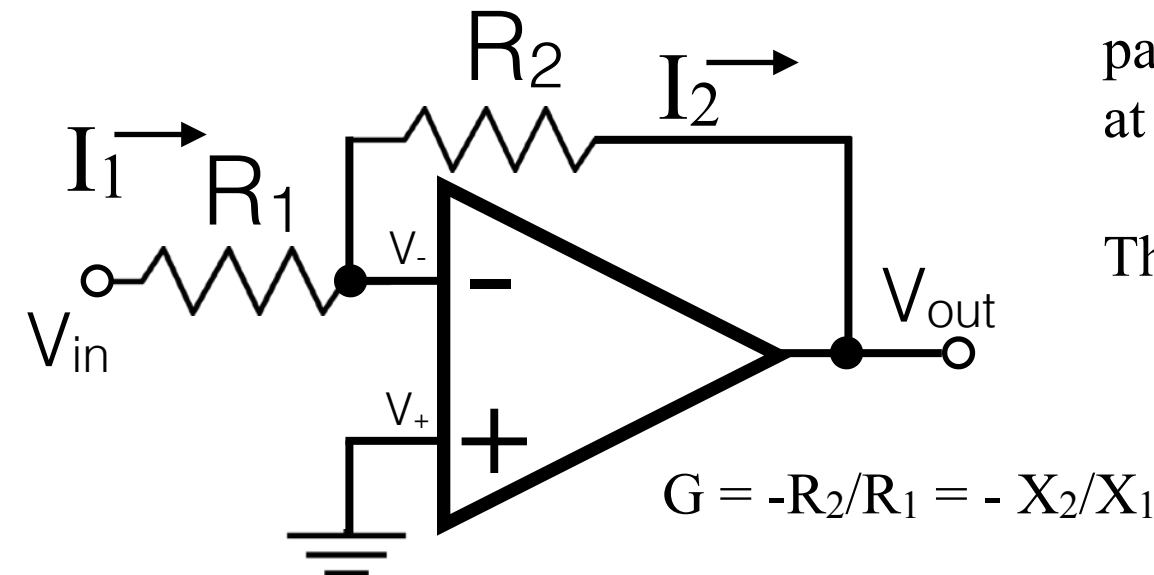


Can also think of this in terms of complex impedance, where it is an inverting amplifier with gain

$$G = -X_R/X_C = -R/(-j/\omega C)$$
$$G = j\omega RC$$

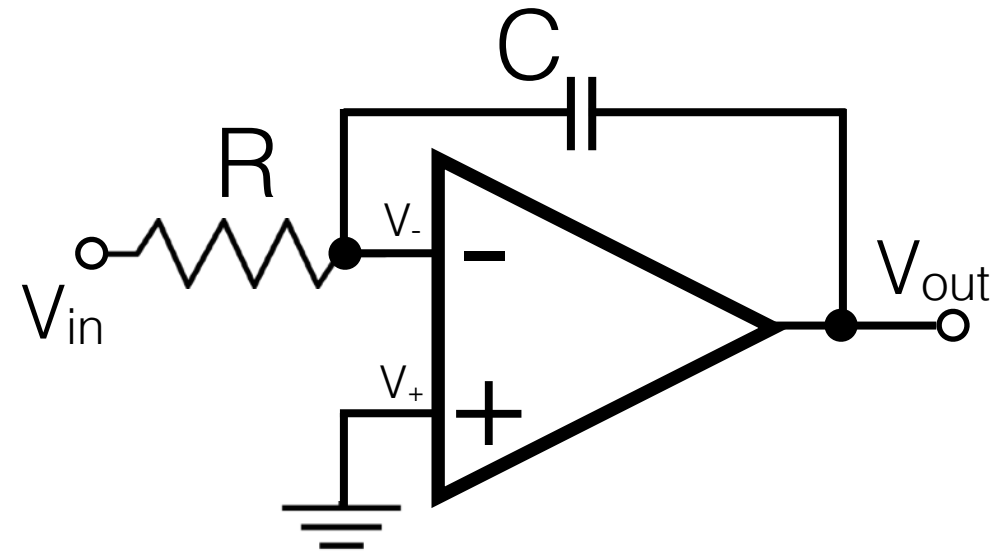
The gain increases with frequency, while the simple, passive differentiator leveled off at high frequency.

The j indicates a phase shift.



Op-amp integrator

Using the golden rules we can analyze the circuit for an integrator



Can also think of this in terms of complex impedance, where it is an inverting amplifier with gain

$$G = -X_C/X_R = -(-j/\omega C)/R$$
$$G = j/\omega RC$$

Gain increases for low frequency.