More transistor circuits: current source, PNP, bootstrapping, Ebers-Moll
Review: Transistor rules of operation

1). $V_{BE} = 0.6 \text{ V}$ or the transistor is off
   I.e., $V_B = V_E + 0.6 \text{ V}$
   Once the transistor is on, $\Delta V_B = \Delta V_E$.

2). $I_C = \beta I_B$.
   And by charge conservation $I_E = I_B + I_C$ so $I_E \equiv I_C$

3). $V_{CE} > 0.2 \text{ V}$

With these simple rules we can analyze most transistor circuits. We’ll add some nuance later today.
Review: Emitter follower

This transistor circuit has the output “follow” the input, with a 0.6 V drop.

\[ X_{in} = \beta R_E \]

\[ V_{in} = 4 + 2 \sin \omega t \]

\[ V_{out} = 3.4 + 2 \sin \omega t \]
Review: Emitter follower

We can remove the clipping at 0 V by setting $V_{EE}$ to a negative supply.

$$X_{in} = \beta R_E$$

$$V_{CC} = +5 \text{ V}$$

$$V_{in} = 2.2 + 2 \sin \omega t$$

$$V_{out} = 1.6 + 2 \sin \omega t$$

$$V_{EE} = -5 \text{ V}$$

Output clips at $V_{CC}$ and 0.6 V above $V_{EE}$. 
Review: Common-emitter amplifier

We can use the current amplification of the transistor to get voltage amplification.

\[ \Delta V_E = \Delta V_B = \Delta V_{in} \]

\[ I_E \approx I_C \]

\[ \Delta V_{out} = - \Delta V_E \left( \frac{R_C}{R_E} \right) = - \Delta V_{in} \left( \frac{R_C}{R_E} \right) \]

Gain = \( \frac{\Delta V_{out}}{\Delta V_{in}} = - \frac{R_C}{R_E} \)
Review: Common-emitter amplifier input biasing

Apply an input bias that puts the emitter close to $V_{EE}$, within a $\Delta V$ that defines the max input swing.

![Circuit diagram]

$V_{CC} = +5 \text{ V}$

Bias the input to put $V_E$ near $V_{EE}$ and $V_{out}$ half way between $V_{CC}$ and $V_{EE}$. *Decoupling capacitor* on the input and output.

Gain = -2, with $R_C = 2k$ and $R_E = 1k$.

This works without any clipping.
Common-emitter amplifier input biasing

Apply an input bias that puts the emitter close to $V_{EE}$, within a $\Delta V$ that defines the max input swing.

$V_{CC} = +5 \text{ V}$

$V_{EE} = -5 \text{ V}$

$V_{out} = V_{in} \cdot \frac{R_2}{R_1 + R_2}$

Suppose I want a max input swing of $\pm 0.1 \text{ V}$

Set $V_E$ to vary from -4.8 to -5.0 V, i.e.,

DC set point for $V_E$ is -4.9 V.

DC set point for $V_{in}$ is -4.3 V.

These are called the *quiescent* values, meaning “when quiet, ie without signal”.

Choose $R_1$ and $R_2$ to be a voltage divider setting $V_{in}$ at -4.3 V.

$V_{in} = V_{EE} + (V_{CC} - V_{EE}) \cdot \frac{R_2}{R_1 + R_2}$

-4.3 = -5 + 10 \cdot \frac{1k}{1k+R_2}

$R_1 = 13k$ and $R_2 = 1k$

Or I could use

$R_1 = 130k$ and $R_2 = 10k$

Which choice is better?
Common-emitter amplifier input biasing

Apply an input bias that puts the emitter close to $V_{EE}$, within a $\Delta V$ that defines the max input swing.

Suppose I want a max input swing of $\pm 0.1$ V
Set quiescent points: $V_{E}=-4.9$ V & $V_{in}=-4.3$ V.
$R_1 = 130k$ and $R_2 = 10k$

To avoid having this stage yank the output of the previous stage to a different voltage, we *decouple* the input from this “DC bias voltage” with a “decoupling capacitor”, $C_{in}$.

$R_{in}C_{in}$ make a high-pass filter letting the signal through and blocking the DC offsets.
What is $R_{in}$?
Common-emitter amplifier input biasing

Apply an input bias that puts the emitter close to $V_{EE}$, within a $\Delta V$ that defines the max input swing.

$$V_{CC} = +5 \text{ V}$$

$$V_{EE} = -5 \text{ V}$$

Input impedance is all paths from input to a fixed voltage ($V_{CC}$, $V_{EE}$, or Gnd).

$$R_{in} = R_1 \parallel R_2 \parallel \beta R_E \cong 130k \parallel 10k \parallel \beta R_E \cong R_2.$$  

High-pass filter should have $f_{3dB}\leq$signal frequency range.
For audio signals, that is 20 Hz, so

$$20 = \frac{1}{2\pi}(10k)C$$

$$C \cong \frac{1}{6*120*10k} \cong \frac{1}{1k*10k} = 0.1 \mu F$$
Common-emitter amplifier input biasing

Now we need to pick $R_E$ and $R_C$

The ratio of $R_E$ and $R_C$ is set by the desired gain, and avoiding output clipping.

Choose gain = 10, and max $V_{in} = 0.1$ V. That means $V_{out}$ swings by $\pm1$ V. Then quiescent point for $V_{out}$ to be at least 1 V away from $V_{CC}$ and $V_E$.

But,

$$V_{out} = V_{CC} - V_E\left(\frac{R_C}{R_E}\right) + V_{EE}\left(\frac{R_C}{R_E}\right)$$

only depends on the gain ratio.

$$V_{out} = 5 - (-4.9\times10) - 5\times10$$

$$= 4$$

That works, but just barely.
Common-emitter amplifier input biasing

Now we need to pick $R_E$ and $R_C$

$$V_{CC} = +5 \text{ V}$$

$$V_{EE} = -5 \text{ V}$$

The ratio of $R_E$ and $R_C$ is set by the desired gain, and avoiding output clipping.

Choose gain = 10.

That means $V_{out}$ swings by $\pm1 \text{ V}$.

Then quiescent point for $V_{out}$ to be at least 1 V away from $V_{CC}$ and $V_{E}$.

But,

$$V_{out} = V_{CC} - V_{E} \left( \frac{R_C}{R_E} \right) + V_{EE} \left( \frac{R_C}{R_E} \right)$$

only depends on the gain ratio.

$$V_{out} = 5 - (-4.9 \times 10^{-1}) - 5 \times 10$$

$$= 4$$

That works, but just barely.
Common-emitter amplifier input biasing

Now we need to pick $R_E$ and $R_C$

$V_{CC} = +5\text{ V}$

$V_{EE} = -5\text{ V}$

Gain of 40 also works, but only if we change the quiescent $V_{out}$ to 1 V.
Common-emitter amplifier input biasing

The challenge here is that $R_E$ affects both the gain and the quiescent $V_{out}$. A small $R_E$ gives big gain but large $I_E$ which affects quiescent $V_{out}$.

$V_{CC} = +5\, \text{V}$

$V_{EE} = -5\, \text{V}$

We want a large $R_E$ for setting quiescent voltages and a small $R_E$ for setting gain.
Common-emitter amplifier input biasing

The challenge here is that $R_E$ affects both the gain and the quiescent $V_{out}$. A small $R_E$ gives big gain but large $I_E$ which affects quiescent $V_{out}$.

We want a large $R_E$ for setting DC quiescent voltages and a small $R_E$ for setting AC gain.

$V_{CC} = + 5 \text{ V}$

$V_{EE} = - 5 \text{ V}$

\[ V_{in} \quad R_1 \quad R_C \quad V_{out} \quad \text{transistor} \quad C \quad B \quad E \quad R_E \quad V_{EE} \]
Common-emitter amplifier input biasing

The challenge here is that $R_E$ affects both the gain and the quiescent $V_{out}$. A small $R_E$ gives big gain but large $I_E$ which affects quiescent $V_{out}$.

Choose $R_1$ and $R_2$ for quiescent $V_E = -4$ V. Choose $R_E = 10k$ and $R_C = 100k$ for quiescent $V_{out} = 1$ V and base gain of 10.

Gain = -10, with $R_C = 100k$, $R_E = 10k$, $R_G = \infty$

$|V_{in}| = 0.1$ V
Common-emitter amplifier input biasing

The challenge here is that $R_E$ affects both the gain and the quiescent $V_{out}$. A small $R_E$ gives big gain but large $I_E$ which affects quiescent $V_{out}$.

Choose $R_1$ and $R_2$ for quiescent $V_E = -4$ V.

Choose $R_E = 10k$ and $R_C = 100k$ for quiescent $V_{out} = 1$ V and base gain of 10.

Gain = -10, with $R_C = 100k$, $R_E = 10k$, $R_G = \infty$

$|V_{in}| = 0.3$ V
Common-emitter amplifier input biasing

The challenge here is that $R_E$ affects both the gain and the quiescent $V_{out}$. A small $R_E$ gives big gain but large $I_E$ which affects quiescent $V_{out}$.

Choose $R_1$ and $R_2$ for quiescent $V_E = -4$ V. Choose $R_E = 10k$ and $R_C = 100k$ for quiescent $V_{out} = 1$ V and base gain of 10.
Common-emitter amplifier input biasing

The challenge here is that $R_E$ affects both the gain and the quiescent $V_{out}$. A small $R_E$ gives big gain but large $I_E$ which affects quiescent $V_{out}$.

Choose $R_1$ and $R_2$ for quiescent $V_E = -4$ V. Choose $R_E = 10k$ and $R_C = 100k$ for quiescent $V_{out} = 1$ V and base gain of 10.

Choose $R_1$ and $R_2$ for quiescent $V_E = -4$ V. Choose $R_E = 10k$ and $R_C = 100k$ for quiescent $V_{out} = 1$ V and base gain of 10.

Gain = -10, with $R_C = 100k$, $R_E = 10k$, $R_G = \infty$

$|V_{in}| = 0.1$ V

Gain = 10

Go back to small input signal; now reduce $R_G$. 

$V_{CC} = +5$ V

$V_{EE} = -5$ V

$V_{in}$

$C_{in}$

$V_{out}$

$R_1$

$R_C$

$B$

$C$

$E$

$R_2$

$R_E$

$R_G$

$C_g$
Common-emitter amplifier input biasing

The challenge here is that $R_E$ affects both the gain and the quiescent $V_{out}$. A small $R_E$ gives big gain but large $I_E$ which affects quiescent $V_{out}$.

Choose $R_1$ and $R_2$ for quiescent $V_E = -4$ V. Choose $R_E = 10k$ and $R_C = 100k$ for quiescent $V_{out} = 1$ V and base gain of 10.

\[
\begin{align*}
V_{CC} &= +5 \text{ V} \\
V_{EE} &= -5 \text{ V}
\end{align*}
\]

\[
\begin{align*}
V_{in} &\quad \text{C_in} \\
R_1 &\quad R_C \\
R_2 &\quad R_E \\
R_G &\quad C_g \\
V_{out} &\quad B \\
E &\quad C
\end{align*}
\]

Gain = -10, with $R_C = 100k$, $R_E = 10k$, $R_G = 3.3k$

$|V_{in}| = 0.1$ V, Gain = 30
Common-emitter amplifier input biasing

The challenge here is that $R_E$ affects both the gain and the quiescent $V_{out}$. A small $R_E$ gives big gain but large $I_E$ which affects quiescent $V_{out}$.

Choose $R_1$ and $R_2$ for quiescent $V_E = -4$ V. Choose $R_E = 10k$ and $R_C = 100k$ for quiescent $V_{out} = 1$ V and base gain of 10.

Gain = -10, with $R_C = 100k$, $R_E = 10k$, $R_G = 200$

$|V_{in}| = 0.005$ V  

Gain = 500
Common-emitter amplifier input biasing

Finally, what can we do about the 1 V quiescent offset on $V_{\text{out}}$?

Remove it with a decoupling capacitor.

Gain = -10, with $R_C = 100k$, $R_E = 10k$, $R_G = 200$

$|V_{\text{in}}| = 0.005 V$  
Gain = 500
Common-emitter amplifier input biasing

This also works if $V_{EE}$ is ground. We just choose quiescent points. In fact with $V_{EE} = \text{Gnd}$, we must have input biasing.

$V_{CC} = + 5 \text{ V}$

![Circuit diagram]

Gain = -10, with $R_C = 100k$, $R_E = 10k$, $R_G = 250$

$|V_{in}| = 0.005 \text{ V}$

Gain = 400

$V_{out}$

$V_{in}$

$C_{in}$

$R_1$

$R_C$

$V_{out}$

$C_{out}$

$R_2$

$R_E$

$R_G$

$C_g$

$V_{EE}$
Common-emitter amplifier input biasing

Some checks of understanding.

Without DC biasing, what would limit the signal?

What is the output impedance of this circuit?

With \( V_{EE} = \text{Gnd} \), about where should you put the quiescent \( V_{out} \)? Where is the quiescent \( V_{in} \)?

In general, how do you maximize the dynamic range?

What would happen if you set \( R_G = 0 \)?
Common-emitter amplifier operation

The transistor is changing the voltage dropped across it to satisfy the rules of operation.

Increase in $V_{in}$ causes increase in $V_E$
That causes an increase in $I_E$
That causes a decrease in $V_C$
The voltage across the transistor, $V_{CE}$, goes down to compensate.
Review: Common-emitter amp AC gain control

We separated the determination of quiescent points from determination of gain with a gain resistor that only matters for signal because of $C_G$.

Gain = $\Delta V_{\text{out}} / \Delta V_{\text{in}} = - \frac{R_C}{(R_E \parallel (R_G+C_G))}$

Gain $\approx - \frac{R_C}{R_G}$

$R_E$ can be chosen to set $V_{\text{out}}$ quiescent point. It can be large, and so can $R_C$.

$R_G$ then is chosen to set the gain, it can vary without altering quiescent points, and it needn’t be too small.
What is the input impedance of the common-emitter amp?

Follow all paths to fixed voltages.

\[ X_{in} = C_{in} + \{R_1 \parallel R_2 \parallel \beta(R_E \parallel [R_G + C_G])\} \]
\[ \cong 0 + \{R_1 \parallel R_2 \parallel \beta(R_E \parallel [R_G + 0])\} \]
\[ \cong 0 + \{R_1 \parallel R_2 \parallel \beta R_G\} \]
\[ \cong R_2 \parallel \beta R_G \]

Can make \( \beta R_G \) reasonably large.

What about \( R_2 \)?

We can make the bias network have very large impedance with a trick called bootstrapping. It uses the same signal specific impedance trick as \( C_G \) does.
Bootstrapping

Add capacitive feedback from $V_E$ to $V_B$.

Now the input impedance is

$$X_{in} = C_{in} + \frac{\beta}{[R_E \parallel (R_G + C_G) \parallel \{C_b + R_1 \parallel R_2\}] \parallel [R_3 + (R_1 \parallel R_2 \parallel \{C_B + R_E \parallel [R_G + C_G]\})]}$$

$$\approx \frac{\beta}{[R_E \parallel R_G \parallel R_1 \parallel R_2]} \parallel [R_3 + (R_1 \parallel R_2 \parallel R_E \parallel R_G)]$$

$$\approx R_3 + (R_1 \parallel R_2 \parallel R_E \parallel R_G)$$

The bootstrapping trick is to make $R_3$ go to infinity for signal.

This works because $X_{C_b}=0$ for signal and $\Delta V_B = \Delta V_E$ for signal.
Bootstrapping

Add capacitive feedback from $V_E$ to $V_B$.

The input impedance is

$$X_{in} \approx R_3 + (R_1 \parallel R_2 \parallel R_E \parallel R_G)$$

Imagine a small signal $v_{in} = \Delta V_{in}$.
It passes through $C_{in}$ and also through the transistor because $\Delta V_B = \Delta V_E$.

$$v_{in} = v_B = v_E = v_3$$

This means a small signal moves the top and bottom of $R_3$ the same, $v_B = v_3$.

The $\Delta V$ across $R_3$ is 0. So no signal current has to flow through $R_3$ into $R_1$ and $R_2$. The impedance of $R_3 \rightarrow \infty$ making $R_1$ and $R_2$ unimportant for $X_{in}$.

Separate DC and AC response.
Constant current source

We can use a transistor to pull a *constant* specified current through a load.

\[ V_{CC} = +5 \text{ V} \]

To get a constant 1mA flow through \( R_L \), even as \( R_L \) changes, we can set \( R_E \) to 1k and \( V_E \) to 1 V.

That sets the value of \( I_E \), which is equal to \( I_C \), regardless of \( R_L \).

Choose \( R_1 \) and \( R_2 \) to make \( V_B = 1.6 \text{ V} \).
Then \( V_E = 1.0 \text{ V} \).
\( I_E = 1 \text{ mA} \).
\( I_C = 1 \text{ mA} \), regardless of \( R_L \).

This works until \( V_C < V_E + 0.2 \)

*Note that there is no input signal here.*
Constant current source

We can use this to pull a specified current through a load.

To get a constant 1mA flow through $R_L$, even as $R_L$ changes, we can set $R_E$ to 1k and $V_E$ to 1 V. That sets $I_E$ which is equal to $I_C$, regardless of $R_L$.

Choose the zener diode to make $V_B = 1.6$ V. The zener reduces sensitivity to $V_{CC}$ variations.
Constant current source

We can use a transistor to pull a constant specified current through a load. This is actually called a current sink since it pulls current from $R_L$.

If we wanted to push current through $R_L$ with the bottom of $R_L$ connected to ground, then we need a different polarity transistor. PNP vs NPN.

Turns on when $V_E = V_B + 0.6$
Constant current source

We can use a PNP transistor to *push a constant* specified current into a load.

\[ V_{CC} = +5 \text{ V} \]

Now we can switch the location of \( R_L \) and \( R_E \). The base’s bias voltage sets \( R_E \) which sets \( I_E \) and hence \( I_C \).

For 1 mA we could set \( R_E = 1k \) and \( V_E = 4 \text{ V} \). That requires \( V_B = 3.4 \text{ V} \) which we get from \( R_1 \) & \( R_2 \) choice.

\[ 3.4 = 5 \frac{R_2}{(R_1+R_2)} \]
Current limiter

Separate from having a constant current, we often want to limit $I < I_{\text{max}}$.

If $Q_2$’s $V_{BE} < 0.6$ V it turns off, so no current flows through $R_b$ and $Q_1$ has a high $V_b$ and $Q_1$ is on.

If enough current flows to cause the voltage drop across $R_s$ to go above 0.6 V, $Q_2$ turns on and current flows through $R_b$. That reduces the base voltage of $Q_1$, lowering the current through $Q_1$ and hence the current through $R_s$ to turn off $Q_2$. This rapid on/off leads to an equilibrium at the max current of $0.6/R_s$.

I.e., attempts to increase the load current beyond $I_L = 0.6/R_s$ (either by higher $V_{CC}$ or lower $R_L$) will lead to a max current of $0.6/R_s$.

E.g., $R_s = 0.6 \Omega$ limits load current to 1 A.
Ebers-Moll model

The simple transistor rules we have been using aren’t the full picture. Two examples of features it misses.

Gain limit with $R_G=0$.

$I_L$ is temperature dependent.
Ebers-Moll model

Gain limit comes from intrinsic resistance in the transistor.

There is a small current-dependent resistance present even when $R_E=0$. Call it $r_e$. $r_e \approx 25\,\Omega / I[\text{mA}]$
Ebers-Moll model

Gain limit comes from intrinsic resistance in the transistor.

There is a small current-dependent resistance present even when $R_E=0$. Call it $r_e$. $r_e \equiv 25\,\Omega / [\text{mA}]$

And it is temperature dependent.

$$I = I_S \left( e^{V/nV_T} - 1 \right) \quad \text{where} \quad V_T = \frac{k_B T}{e}$$

$V_T \equiv 25\,\text{mV}$ at room temperature.$I_S$ is also temperature dependent.
This illustrates something called transconductance, which is like gain but unit-full.
Gain = \( \Delta V_{\text{out}} / \Delta V_{\text{in}} \) is unitless.
But really, changing \( V_{\text{in}} \) changes \( V_{BE} \).
That changes \( I_C \) through the Ebers-Moll relation.

\[
I = I_S \left( e^{V/nV_T} - 1 \right)
\]

So the transistor’s “gain” is really
\[
g_m = \Delta I_C / \Delta V_{\text{in}}.
\]

This is called transconductance because conductance is 1/resistance, and the sub-m is short for mho, which is opposite of ohm.

The \( R_C \) and \( R_E \) used in a common emitter amplifier convert that \( \Delta I_C \) back into a \( \Delta V_{\text{out}} \).
Ebers-Moll model & transconductance

\[ I = I_S \left( e^{V/nV_T} - 1 \right) \approx I_S e^{V_{BE}/nV_T} \]

Strong function of \( V_{BE} \), e.g.,
\[ \Delta V_{BE} = 18 \text{ mV doubles } I_C. \]
\[ \Delta V_{BE} = 60 \text{ mV increases } I_C \text{ by x}10. \]

We can see where \( r_e \) comes from by calculating \( dI/dV \).

\[ 1/r_e = dI/dV = (1/nV_T) I_s e^{V_{BE}/nV_T} = I/nV_T \]

\[ r_e = nV_T/I = 25 \text{ mV}/I = 25\Omega \text{ / } I[\text{mA}] \text{ at room temperature} \]
Ebers-Moll model & temperature effects

\[ I = I_s \left( e^{\frac{V}{nV_T}} - 1 \right) \approx I_s \frac{V_{BE}}{nV_T} = I_s e^{\frac{V_{BE}}{kBT}} \]

We can see the effect of temperature on our current mirror.

Set \( V_B \approx 5.6 \), so \( V_E = 5.0 \) V and \( I_E = 1 \) mA.

If temperature increases, \( I_C = I_E \) reduces.

That reduces \( V_E \)

which increases \( V_{BE} \)

which increases \( I_C \).

So we have “negative feedback” holding the circuit at an equilibrium behavior, insensitive to temperature.

But, \( I_S \) is also temperature dependent, with opposite and stronger dependence, \( \sim 9\%/\degree C \).

So we need to build in more negative feedback.
Current mirror for temperature stability

\[ I = I_s \left( e^{V/nV_T} - 1 \right) \approx I_s e^{V_{BE}/nV_T} \approx I_s e^{V_{BE}q/kBT} \]

Set (program) Ip on left side.
Current mirror for temperature stability

\[ I = I_s \left( e^{V/nV_T} - 1 \right) \approx I_s e^{V_{BE}/nV_T} \approx I_s e^{V_{BE}q/kBT} \]

Set (program) \( I_p \) on left side.

\[ V_B = V_{CC} - 0.6 = V_C \]

\[ I_p = V_C/R_p \]

\( I_{load} = I_p \) because \( V_{BE} \) for \( Q_2 \) is the same as \( V_{BE} \) for \( Q_1 \).
Current mirror for temperature stability

\[ I = I_s \left( e^{V/nVT} - 1 \right) \approx I_s e^{V_{BE}/nVT} \approx I_s e^{V_{BE}q/kBT} \]

Set (program) \( I_p \) on left side.

\[ V_B = V_{CC} - 0.6 = V_C \]
\[ I_p = V_C/R_p \]
\( I_{load} = I_p \) because \( V_{BE} \) for \( Q_2 \) is the same as \( V_{BE} \) for \( Q_1 \).

Use matched transistors:
- same doping concentrations means same \( I_s \), and \( I_s(T) \);
- same substrate means same temperature.

Then they also have the same Ebers-Moll relation.

If some \( \Delta T \) causes \( I_p \) to increase, then \( R_p \) pushes \( V_{BE} \) up to reduce \( I_p \). Same change on both sides.
Differential amplifier

If we try to transmit a signal a long distance, we need to worry about RF pickup because the wires act as an antenna.

We could amplify the signal before transmitting to make it large compared to any pickup. But then it becomes a powerful transmitter causing pickup on other wires nearby.
Differential amplifier

If we try to transmit a signal a long distance, we need to worry about RF pickup because the wires act as an antenna.

We could amplify the signal before transmitting to make it large compared to any pickup. But then it becomes a powerful transmitter causing pickup on other wires nearby.

Best to transmit signals with small signals that are immune to pickup; use low-voltage differential signals (LVDS) on twisted pairs of wires.
Differential amplifier

\[
\text{Signal}^+ = +\Delta V/2 \\
\text{Signal}^- = -\Delta V/2
\]
Differential amplifier
Analyze this by 1st calculating $V_A$.

$$V_A = V_{EE} + I_{EE}R_{EE}$$

$$I_{EE} = I_{E1} + I_{E2}$$

$$= (V_{E1} - V_A)/R_E + (V_{E2} - V_A)/R_E$$

$$= (V_{E1} + V_{E2})/R_E - 2V_A/R_E$$

$$V_A = V_{EE} + R_{EE}/R_E (V_{E1} + V_{E2})/R_E - 2R_{EE}V_A/R_E$$

$$V_A = \frac{R_E V_{EE} + R_{EE} (V_{E1} + V_{E2})}{R_E + 2R_{EE}}$$

$$\Delta V_A = (\Delta V_{E1} + \Delta V_{E2}) \frac{R_{EE}}{R_E + 2R_{EE}}$$

If $\Delta V_{E1} = - \Delta V_{E2}$ then $\Delta V_A = 0$
Differential amplifier

Analyze this by 1st calculating $V_A$.

\[ V_A = V_{EE} + I_{EE}R_{EE} \]

\[ I_{EE} = I_{E1} + I_{E2} = (V_{E1} - V_A)/R_E + (V_{E2} - V_A)/R_E = (V_{E1} + V_{E2})/R_E - 2V_A/R_E \]

\[ V_A = V_{EE} + R_{EE}/R_E (V_{E1} + V_{E2})/R_E - 2R_{EE}V_A/R_E \]

\[ V_A = \frac{R_EV_{EE} + R_{EE}(V_{E1} + V_{E2})}{R_E + 2R_{EE}} \]

\[ \Delta V_A = (\Delta V_{E1} + \Delta V_{E2}) \frac{R_{EE}}{R_E + 2R_{EE}} \]

If $\Delta V_{E1} = -\Delta V_{E2}$ then $\Delta V_A = 0$

This makes the right side just a common-emitter amp with $v_{out} = (-R_C/R_E) v_2$

If $v_2 = -\Delta V_{in}/2 = -v_{in}/2$ then $v_{out} = (R_C/R_E)v_{in}$. 
Now consider the *common mode* signal, where $v_1 = v_2 = \bar{v} = v_{CM}$

That makes $\Delta I_{E1} = \Delta I_{E2}$ & $\Delta I_{EE} = 2\Delta I_{E1}$

Written with “variation notation” its $i_{E1} = i_{E2}$ and $i_{EE} = 2i_{E1}$

So, $\Delta V_A = v_A = i_{EE}R_{EE} = 2i_{E1}R_{EE}$

Now use Ohm’s law to find $i_{E1}$ as

$$i_{E1} = \frac{(v_E - v_A)}{R_E} = \frac{(v_{CM} - 2i_{E1}R_{EE})}{R_E}$$

So,

$$i_{E1} = \frac{v_{CM}}{(R_E + 2R_{EE})}$$

Common mode gain $= -\frac{R_C}{(R_E + 2R_{EE})}$

Differential gain $= -\frac{R_C}{2R_E}$

$$v_{out} = -i_{E1} \frac{R_C}{-v_{CM}} \frac{R_C}{(R_E + 2R_{EE})}$$